Department of Mathematics, Indian Institute of Technology Bombay
SI 507: Numerical Analysis
Course Information

S. Sivaji-Ganesh **Autumn 2016**

1. General Information

§ **Location and Timings** Classes will be held in **Slot 6A, 6B, 12B** (in Room no. 114), Mathematics department. There will some lab sessions throughout the semester using MATLAB.

§ **Assistance** My office is located on the second floor (Room no. 210D) of Mathematics department building. You can meet me whenever you find me in my office for any assistance regarding the course. However it is preferable that we meet after fixing-up a mutually convenient time.

§ **Attendance** Currently 80% attendance in the classes is compulsory, failing which you will be awarded a **DX** grade. It is your duty to make sure that you fulfill the attendance requirement. In case you fall short of 80% attendance, and you are absent for X% of classes where $X > 20$, then you must have justification for all the X% absence.

2. Texts and Syllabus

§ **Textbook**

(i) S. Baskar and S. Sivaji Ganesh, *Introduction to numerical analysis.*

You **MUST** have a copy of the lecture notes with you. Tutorial problems are also included in these notes.

§ **Topics to be covered(not necessarily in the same order)**

(1) **Mathematical Preliminaries** Review of Continuity, Differentiation, Taylor's theorem, Orders of convergence.

(2) **Error Analysis** Floating-point approximation, arithmetic using n-digit rounding and chopping; types of errors; loss of significance; propagation of relative error in arithmetic operations, stable computations.

(3) **Numerical Linear Algebra**

Solution of simultaneous linear equations: Gaussian elimination, pivoting, LU decomposition; Norms of vectors and matrices, Gauss-Seidel, Jacobi methods; Steepest descent method, conjugate gradient method.

Eigenvalue problems: Power method, QR decomposition; Schur's and Gershgorin's theorems.

(4) **Solution of Nonlinear Equations** Bisection, regula-falsi, Newton's and secant methods; Order of convergence; fixed point method; Error estimation and stopping criteria.

(5) **Interpolation of Functions by Polynomials** Lagrangian form of interpolation polynomial; Error estimation; Newton's form, divided differences; Chebyshev interpolation points, Hermite interpolation, Spline interpolation, Cubic splines.

(6) **Numerical Differentiation and Numerical Integration** Numerical approximation of derivatives, order of approximation; Newton-Cotes formulas (rectangle, trapezoid, and Simpson's rules); Order of numerical quadrature, error estimates; Composite rules; Acceleration of convergence: Romberg's method.

(7) **Numerical Solution of Ordinary Differential Equations** Euler method, Runge-Kutta methods, Multistep methods, Local and Global errors: Stability.

3. Relevance of the course

§ **What is Numerical analysis?**

In an essay by L.N. TREFETHEN entitled "The definition of Numerical analysis" which appears as an appendix in the book *Numerical Linear algebra* authored by Trefethen and Bau, the author says that the impression one gets about the subject of Numerical analysis on opening an arbitrary book on Numerical analysis is that it is about study of "roundoff errors," which arise due to the floating point arithmetic used in Computer. But this is a wrong impression. Trefethen proposes a definition: *"Numerical analysis is the study of algorithms for the problems of continuous mathematics"*. We assent this definition and illustrate with very simple problems and algorithms for their solution.

§ **Why should we learn Numerical analysis?**

With availability of more and more efficient computers and softwares like Matlab, Maple, one might feel that there is no need to study simple numerical methods for solving linear, nonlinear equations, ODEs and PDEs, since softwares like Matlab have built-in programs that do the job. But they do not reveal the algorithms that are used, thereby we do not know the limitations, complexities involved in those programs; one has to be careful with anything that solves all problems! This is precisely the reason for us to learn and design our own algorithms where we are aware of what happens while computing in a computer/calculator; which allow us to modify or change the strategy to solve a problem numerically.

§ **Applications of Numerical analysis?**

Numerical methods are helpful in integrating functions whose anti-derivatives are not known. Even for well-posed problems involving differential equations, explicit analytic solutions are not known; and numerical methods are also useful in their computation. The importance of Numerical methods in different fields of engineering is highlighted in the online text available at *http://numericalmethods.eng.usf.edu/physical_problems_text.html*

§ **How to do Numerical analysis?**

A course on numerical analysis is incomplete without implementing the algorithms ourselves on a computer. We will learn the implementation of some algorithms that solve some simple problems using MATLAB.

§ **Course objectives**

(i) To understand numerical methods and related errors in solving nonlinear equations, ODEs and PDEs, systems of linear equations.

(ii) To understand the subtleties arising in computer implementation of several algorithms.

(iii) To give an exposure to MATLAB language.

4. Evaluation and Grading

Examinations: There will be 4 examinations including Mid-sem, and a lab examination along with the End-sem. Dates for quizzes will be announced in the class.

S.No.	Assessment details	Total marks
1	Lab Examination	15
2	Best 3 out of total 4 Examinations	45
3	Endsemester Exam	40
5	Class participation	+5

Grading Scheme: The grading will be **ABSOLUTE**. Please note that for passing this course, it is necessary to obtain a minimum of 35 marks. I will follow the grading scheme given below.

Marks range	35-39	40-49	50-59	60-69	70-79	80-100
Grade	CD	CC	BC	BB	AB	AA

5. Other information

Copying in assignments and examinations: This is a serious offence. Any student found copying will be reported for disciplinary action.