Session for Young Researchers in Commutative Algebra and Algebraic Geometry

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Abstracts of talks

Smoothness of limiting rank 4 Azumaya algebra structures and consequences

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We outline the results of the recent paper "Limits of Rank 4 Azumaya Algebras and Applications to Desingularisation", Proc. Indian Acad. Sci., (Math. Sci.), Vol. 112, No. 4, November 2002, pp. 485–537, and in addition another new application of the central result of Part A of that paper to computing the groups of self-similitudes and self-isometries of a rank 3 quadratic module (the quadratic form need not be 'good' and the base ring can be arbitrary) generalising well-known theorems for 'good' quadratic forms.

Maximal tori determining algebraic groups

Shripad M. Garge, Harish-Chandra Research Institute, Allahabad

Let H be a connected, reductive algebraic group defined over a field k. It is natural to ask whether H is determined by the set of k-isomorphism classes of maximal k-tori in it. We study this question over global fields, local non-archimedean fields and finite fields. We have following results.

Theorem 1 *Let k be a finite field, a global field or a local non-Archimedean field. Let H_1 and H_2 be two split, connected, reductive algebraic groups defined over k. Suppose that for every maximal k-torus $T_1 \subset H_1$ there exists a maximal k-torus $T_2 \subset H_2$, such that the tori T_1 and T_2 are k-isomorphic and vice versa. Then the Weyl groups $W(H_1)$ and $W(H_2)$ are isomorphic. Moreover, if we write the Weyl groups $W(H_1)$ and $W(H_2)$ as a direct product of the Weyl groups of simple algebraic groups, $W(H_1) = \prod_{\Lambda_1} W_{1,\alpha}$, and $W(H_2) = \prod_{\Lambda_2} W_{2,\beta}$, then there exists a bijection $i : \Lambda_1 \to \Lambda_2$ such that $W_{1,\alpha}$ is isomorphic to $W_{2,i(\alpha)}$ for every $\alpha \in \Lambda_1$.*

Theorem 2 *Let k be as in the previous theorem. Let H_1 and H_2 be two split, connected, semisimple algebraic groups defined over k with trivial center. Write H_i as a direct product of simple groups, $H_1 = \prod_{\Lambda_1} H_{1,\alpha}$, and $H_2 = \prod_{\Lambda_2} H_{2,\beta}$. If the groups H_1 and H_2 satisfy the condition given in the above theorem, then there is a bijection $i : \Lambda_1 \to \Lambda_2$ such that $H_{1,\alpha}$ is isomorphic to $H_{2,i(\alpha)}$, except for the case when $H_{1,\alpha}$ is a simple group of type B_n or C_n, in which case $H_{2,i(\alpha)}$ could be of type C_n or B_n.*

The proof uses elaborate information available about the conjugacy classes in Weyl groups of simple algebraic groups. Our analysis finally depends on the knowledge of characteristic polynomials of elements in the Weyl groups considered as subgroups of $GL_n(\mathbb{Z})$.

A Bound for the Degree

Leah Gold, Texas A & M University, USA

In 1985 Huneke and Miller published a succinct formula for the degree of R/I when R/I is Cohen-Macaulay and has a pure resolution. Later Huneke, Srinivasan, and Herzog generalized this statement with some conjectures about bounds for the degree. Numerous calculations have supported these conjectures, but general proofs are elusive. We will discuss the conjectures and recent progress in enumerating specific cases where the conjectures are true.

Rees algebras of modules over a regular local ring

Futoshi Hayasaka, Meiji University, Japan

Let $(A, \mathfrak{m})$ be a regular local ring and let $M \subseteq F = A^d$ be the second syzygy module of the residue field $A/\mathfrak{m}$. Then the Rees algebra $\mathcal{R}(M)$ of M is defined to be the image of the natural homomorphism from the symmetric algebra of M to the symmetric algebra of F. In this talk, we proove the following result: the Rees algebra $\mathcal{R}(M)$ of M is a Gorenstein UFD. (Joint work with S. Goto, K. Kurano and Y. Nakamura.)

Projective normality of abelian varieties

Jaya N. Iyer, Institute of Mathematical Sciences, Chennai

Let A be a abelian variety over complex numbers of dimension g. Suppose A is simple, i.e., A is not isogenous to a product of abelian varieties of smaller dimension and L be an ample line bundle on A. If $\dim H^0(A, L) > 2^g \cdot g!$, then L gives a projectively normal embedding, for any $g > 0$. We will indicate a proof of the above property of L, usually called as the N_0-property.

Complete intersection ideals and a question of Nori

Manoj K. Keshari, Tata Institute of Fundamental Research, Mumbai

Let A be a regular affine domain of dimension d over an infinite perfect field k and let n be an integer such that $2n \geq d+3$. Let I be an ideal of $A[T]$ of height n. Assume that $I = (f_1, \ldots, f_n) + I^2$ and $I(0) = (a_1, \ldots, a_n)$ with $f_i(0) = a_i$ for $1 \leq i \leq n$, where $I(0)$ is an ideal of A generated by $f(0)$ such that $f(T) \in I$. In this talk, we will discuss the following question of Nori: Is $I = (g_1, \ldots, g_n)$ with $f_i - g_i \in I^2$ and $g_i(0) = a_i$ for $1 \leq i \leq n$, which Bhatwadekar and myself answered in affirmative.

Computation of Toric Residues

Amit Khetan, University of Massachusetts, Amherst, USA

The toric residue is a function of $n+1$ divisors on an n-dimensional toric variety. They have been found to be useful in a variety of contexts such as mirror symmetry, the Hodge structure of hypersurfaces, the study of sparse resultants, and in the theory of A-hypergeometric systems. In joint work with Ivan Soprounov, I present techniques towards finding an explicit matrix representation of an element of toric residue 1, extending results by Cattani, Cox, and Dickenstein, The tools are a mixture of combinatorics, commutative algebra, and algebraic geometry.

An application of classical invariant theory to distinguishing isomorphism classes of algebraic objects

Vijay Kodiyalam, Institute of Mathematical Sciences, Chennai

A generalisation of a theorem of Procesi and Razmyslov is presented along with some applications to distinguishing (a) semisimple Hopf algebras (b) semisimple Lie algebras and (c) planar algebras.

Resolutions of modules over local rings

Tony. J. Puthenpurakal, Indian Institute of Technology Bombay, Mumbai

We report on joint work with (i) S. Iyengar, and (ii) J. Asadollahi, concerning resolutions of modules. A common feature will be the use of techniques from the theory of Hilbert functions of modules over local rings, namely Ratliff-Rush filtrations, superficial elements and reductions.

Let $(A, \mathfrak{m})$ be a local Noetherian ring and let M be a finite A -module of positive depth. In [1] we prove that if $n > \operatorname{reg}(M)$ and $\operatorname{injdim}_A M/\mathfrak{m}^{n+1}M < \infty$, then A is a regular or a hypersurface ring. We also study the growth of the function $n \mapsto \lambda\big(\operatorname{Tor}_1^A(M, A/\mathfrak{m}^{n+1}A)\big)$.

In a recent paper of Goto and Hayasaka [3], applications of $\mathfrak{m}$-full ideals to some homological questions was first studied. In [2] we use techniques in [1] and [3], to give an analogue of a result due to Levin and Vasconcelos [4].

[1] S. Iyengar and T. J. Puthenpurakal, Finite Homological dimensions and the associated graded module, *Preprint*, (2002).

[2] J. Asadollahi and T. J. Puthenpurakal, Applications of $\mathfrak{m}$-full ideals in the study of resolutions, *Preprint*, (2003).

[3] S. Goto and F. Hayasaka, Finite homological dimension and primes associated to integrally closed ideals, *Proc. Amer. Math. Soc.* **130** (2002), 3159-3164.

[4] G. Levin and W. V. Vasconcelos, Homological dimensions and Macaulay rings, *Pacific J. Math* , **46** (2000), 315-323.

Hilbert functions of points on Schubert varieties of maximal isotropic subspaces

K. N. Raghavan, Institute of Mathematical Sciences, Chennai

This is a report on joint work with Sudhir Ghorpade. We calculate, in the spirit of Kreiman and Lakshmibai, Hilbert functions of points on Schubert varieties of maximal isotropic subspaces. Specializing to the case when the point is the "identity coset", we recover results of Conca on symmetric determinantal varieties.

K-theory of toric bundles

V. Uma, Institute of Mathematical Sciences, Chennai

Let $p : E \longrightarrow B$ be a principal bundle with fibre and structure group the torus $T \cong (\mathbb{C}^*)^n$ over a topological space B. Let X be a nonsingular projective T-toric variety. One has the X-bundle $\pi : E(X) \longrightarrow B$ where $E(X) = E \times_T X$, $\pi([e, x]) = p(e)$. This is a Zariski locally trivial fibre bundle in case $p : E \longrightarrow B$ is algebraic.

This talk is based on a recent paper with Parameswaran Sankaran titled "Cohomology of toric bundles" where we describe (i) the singular cohomology ring of $E(X)$ as an $H^*(B; \mathbb{Z})$-algebra, (ii) the topological K-ring of $K^*(E(X))$ as a $K^*(B)$-algebra when B is compact. When $p : E \longrightarrow B$ is algebraic over an irreducible, nonsingular, noetherian scheme over $\mathbb{C}$, we describe (iii) the Chow ring of $A^*(E(X))$ as an $A^*(B)$-algebra, and (iv) the Grothendieck ring $\mathcal{K}^0(E(X))$ of algebraic vector bundles on $E(X)$ as a $\mathcal{K}^0(B)$-algebra.

In this talk we shall concentrate on the structure of the ring $\mathcal{K}^0(E(X))$.