The Mathematics Consortium

BULLETIN

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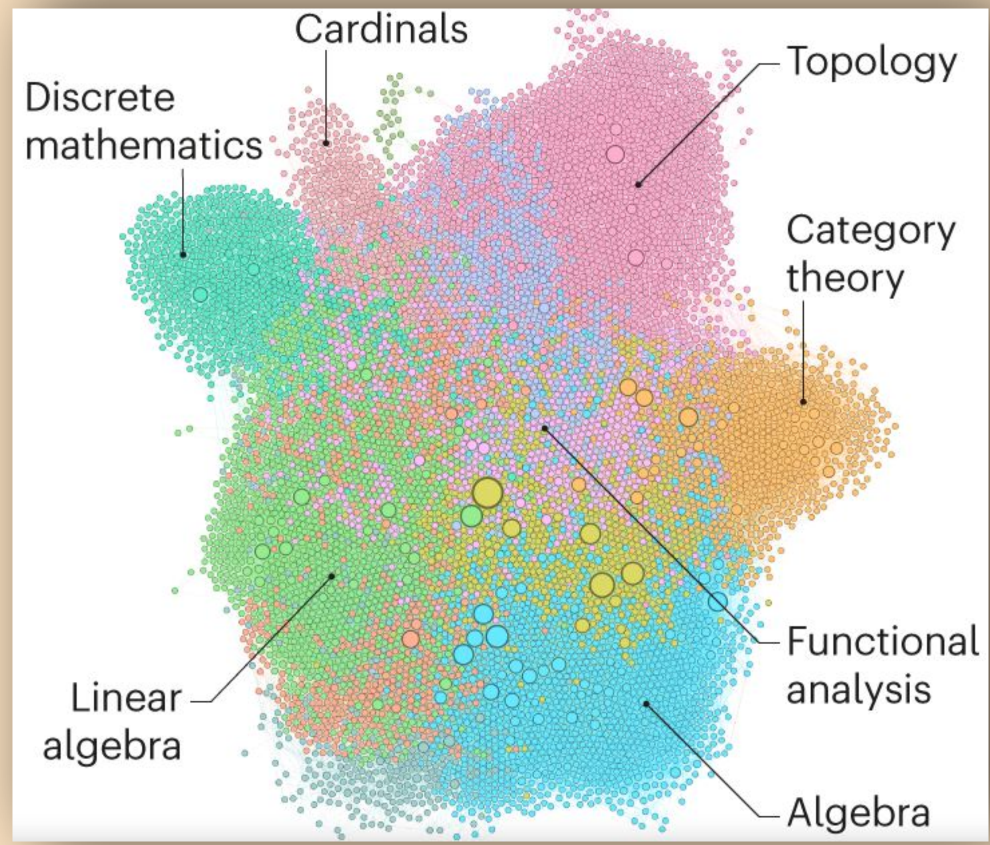
A proof-assistant software Lean



Typical output of Lean in the form of a complex Network

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About the Cover-page: *In the proof-assistant package Lean, users enter mathematical statements based on simpler statements and concepts that are already in the Lean library. The output, seen here in the case of Scholze and Clausen's key result, is a complex network. The statements have been colour-coded and grouped by subfield of mathematics.*
Credit: Patrick Massot
Source: https://www.nature.com/articles/d41586-021-01627-2

Typeset and designed by: Praanjala Typesettings, Contact: +91 99603 03874, Email: mr.texdoc@gmail.com.

inauguration was rather high and we were sure that he would not be able to climb the steps to receive the medal from the President. The President's security had given the organizers permission to seat 9 people on the dais. Since the prize winners were only revealed on the morning of the inauguration itself, this was not a problem we could have anticipated. The security were rather rigid about this and we knew that if we ask them at the last minute to make an exception and allow Prof. Nirenberg to sit on the dais too, they would decline us permission. Luckily for us the President's husband who had been invited by us on Rashtrapati Bhavan's request was indisposed and did not come. Prof. Raghunathan immediately decided that we would seat Prof. Nirenberg on the dais. There would be 9 people on the dais and the security would be none the wiser.

So were the aims of Prof. Raghunathan fulfilled. Fifteen hundred Indian mathematicians attended the ICM, a thousand funded directly by the EOC and 500 by the DST. About a 100 carefully selected senior school students participated as volunteers. There were two 'public lectures' attended by over 2500 school students - one on The Proof by Gunter Ziegler and the other by Bill Barton titled 'Where is mathematics taking me, an exciting ride into the future'. The jury is out. We tried.

It was surely a privelege for me to work with Professor Raghunathan.

□ □ □

5. A Review on a Book

A Course in Calculus and Real Analysis (Second Edition),
By Sudhir R. Ghorpade and Balmohan V. Limaye (Springer, 2018)

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Calculus of a single variable is a basic course which is in the curriculum of most of the undergraduate students in science and engineering. This is also one of the fundamental courses for the undergraduate students in mathematics. Prior to entering an undergraduate program, these students have already been introduced to most of the standard topics in calculus of a single variable, such as limit, continuity, differentiation and integration in their high school curriculum. They have also been exposed to some of the applications of differentiation and integration. Sequences and series of real numbers are also introduced at this level to some extent. For instance, exponential functions are defined in terms of power series in high school. However, at the school level, the concepts mentioned above are neither introduced nor defined in a mathematically rigorous manner. Further, most of the associated results are also not proved at this stage.

Calculus at the undergraduate level commences as a continuation of the calculus taught at high school. However, the manner in which the subject is approached and the syllabus can vary vastly as they depend on the curriculum and the instructors teaching the course. Catering to a variety of needs, there are a large number of books already available on calculus. Some books assume the concepts and the results taught at the school level and principally deal with the applications of the concepts. Of course, the applications of calculus are important for scientists and engineers. But, a significant component of education is the training of the mind for precise and rigorous thinking. To achieve this, it is imperative that the student is exposed to the basic results and their proofs. Further, a large number of undergraduate students are also desirous of knowing the genesis of concepts, proofs and their applications. Hence, many authors of the books on calculus attempt to maintain a balance between mathematical rigour and applications. For instance, some books present the statement of the Bolzano-Weierstrass theorem without a proof but using this result, they develop the requisite analysis for calculus. Some present the completeness property of the real number system as an axiom and prove even the Bolzano-Weierstrass theorem. The same books may avoid the proofs of some important results in calculus such as the L'Hospital rule, Pappus theorems, convergence of Newton-Raphson method etc.

We rarely come across a book on calculus of a single variable which is all inclusive and presents the proofs of all the statements presented in the book. The book under the review is one such book. This book is not only self contained, but its approach is also mathematically rigorous. The book even outlines the construction of real number system in the Appendix. This book does not even assume the area of the circle for measuring the angles in terms of radians and also for defining the trigonometric functions.

The topics in a standard course of calculus of a single variable are covered in the first nine chapters in this book. The tenth chapter discusses sequences and series of realvalued functions of a real variable which is typically not addressed in a standard text on calculus. Concepts and results are explained and illustrated with numerous examples and geometric figures. The book contains a large number of problems. Geometric interpretations of various results, concepts and methods are provided wherever possible. This will enable the student to visualize the ideas. All the steps of the proofs and illustrated examples are explained very clearly. The exposition presented in the "Notes and comments" can be inspiring for students.

The presentation of the topics covered in this book is unusual in comparison to a standard textbook on calculus. For instance, the trigonometric functions are introduced in Chapter 7. Examples using these functions are also avoided in the preceding chapters, where the concepts of continuity and differentiability are discussed. Trigonometric functions are typically used in many standard examples and counterexamples, when the concepts of limit, continuity and differentiability are discussed. However, the reasons for avoiding the trigonometric functions till Chapter 7 are explained. In a standard textbook of calculus, the geometric properties such as the intermediate value property, monotonicity, convexity and local extrema are discussed after the introduction of the concepts of either continuity or differentiability. This book does this differently.

When the undergraduate students are taught calculus in a mathematically rigorous manner, they will encounter the proofs of the results, perhaps already known to them, and the logical reasoning behind them for the first time. At this juncture, it is natural for mathematically inclined students to ask a lot of questions. Answers for many such questions can be found in this book. Moreover, the book has many interesting results in every topic, which are usually not found in standard text books. Hence this book can be an excellent reference book for the instructors, mathematically inclined students who are doing a course on calculus and undergraduate students in mathematics. This book can also be used by students who have already done a course on calculus or real analysis for consolidating their knowledge. However, it is difficult to adopt this book as a text book for introductory courses in either calculus or real analysis.

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No matter how correct a mathematical theorem may appear to be, one ought never to be satisfied that there was not something imperfect about it until it also gives the impression of being beautiful. — George Boole — Quoted in D MacHale, Comic Sections (Dublin 1993).
Source: *https://mathshistory.st-andrews.ac.uk/Biographies/Boole/quotations/*

If one has really technically penetrated a subject, things that previously seemed in complete contrast, might be purely mathematical transformations of each other. – John Von Neumann
Source: *https://en.wikiquote.org/wiki/John_von_Neumann*