Indian Institute of Technology Bombay
MA 001 Preparatory Mathematics I

Autumn 2012 SRG

Exercise Set 6

1. In each of the following cases, find the equation of the line passing through the given two points.

   (i) $(1, -1)$ and $(2, -3)$, (ii) $(1, 7)$ and $(3, -4)$, (iii) $(3, -2)$ and $(5, -1)$.

2. Let L_1 and L_2 be lines in the plane $\mathbb{R}^2$ defined by

$$A_1x + B_1y + C_1 = 0 \quad \text{and} \quad A_2x + B_2y + C_2 = 0,$$

   respectively. Show that L_1 and L_2 are parallel if and only if

$$\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} := A_1B_2 - A_2B_1 = 0.$$

   Further show that if L_1 and L_2 are not parallel, then they intersect in a unique point, (x_0, y_0), which is given by

$$x_0 = \frac{B_1C_2 - B_2C_1}{A_1B_2 - A_2B_1} \quad \text{and} \quad y_0 = \frac{A_2C_1 - A_1C_2}{A_1B_2 - A_2B_1}.$$

3. In each of the following cases, find the points of intersection of the given two lines.

   (i) $2x - 5y + 1 = 0$ and $x + y + 4 = 0$, (ii) $ax + by - 1 = 0$ and $bx + ay - 1 = 0$.

4. Show that any three points $P_1 = (a_1, b_1)$, $P_2 = (a_2, b_2)$ and $P_3 = (a_3, b_3)$ in the plane $\mathbb{R}^2$ are collinear if and only if

$$\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} := a_1b_2 - a_1b_3 - a_2b_1 + a_3b_1 + a_2b_3 - a_3b_2 = 0.$$

5. In each of the following cases, determine if the given three points are collinear. If yes, then find the equation of the line passing through them.

   (i) $(1, -3)$, $(-1, -5)$ and $(2, -2)$, (ii) $(4, 3)$, $(-2, 1)$ and $(1, 2)$.

6. Show that for any $\lambda, \mu \in \mathbb{R}$, the points $(\lambda, 2\mu)$, $(3\lambda, 0)$, $(2\lambda, \mu)$ and $(0, 3\mu)$ are collinear.

7. Show that any three distinct, non-parallel lines defined by

$$A_1x + B_1y + C_1 = 0, \quad A_2x + B_2y + C_2 = 0 \quad \text{and} \quad A_3x + B_3y + C_3 = 0,$$

are concurrent (that is, they pass through a common point) if and only if

$$\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} := A_1B_2C_3 - A_1B_3C_2 - A_2B_1C_3 + A_3B_1C_2 + A_2B_3C_1 - A_3B_2C_1 = 0.$$

8. In each of the following cases, determine if the given three lines are concurrent. If yes, then find the common point of intersection.
   (i) $3x - y - 2 = 0$, $5x - 2y - 3 = 0$ and $2x + y - 3 = 0$,
   (ii) $7x - 4y + 1 = 0$, $x - y + 1 = 0$ and $2x - y = 0$.

9. If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are two distinct points, then show that the line joining P_1 and P_2 is parametrically given by

$$x = (1-t)x_1 + tx_2 = x_1 + t(x_2 - x_1) \quad \text{and} \quad y = (1-t)y_1 + ty_2 = y_1 + t(y_2 - y_1).$$

10. If the parametric equations $x = a + bt,\ y = c + dt$ and $x = a' + b't,\ y = c' + d't$ determine the same line, then is it true that (a, b, c, d) is proportional to (a', b', c', d')? Justify your answer.

11. Find the equations of the lines given parametrically by
    (i) $x = 2 + 3t,\ y = -1 + 4t,$ (ii) $x = -3 - t,\ y = 1 - 2t,$

12. Find the equation of the line which passes through the intersection of the lines $3x + 4y - 8 = 0$, $2x - 5y + 3 = 0$ and is perpendicular to the line $x + 2y - 4 = 0$.

13. Find the equation of the lines passing through the point $(4, -3)$ and intercepting on the coordinates axes a triangle whose area is 3 square units.

14. A straight line intercepts congruent segments on the coordinate axes in the first quadrant. Find the equation of this line if the area of the triangle formed by the line with the coordinate axes is equal to 18 square units.

15. In each of the following cases, determine the distance from the point P to the given line. Also, find the "projection" of P on the given line, that is, find the coordinates of the point Q where a perpendicular drawn from P meets the given line.
    (i) $3x + 4y - 5 = 0$ and $P = (2, 6)$, (ii) $5x + 12y - 20 = 0$ and $P = (2, 3)$.

16. Let L_1 and L_2 be parallel lines defined by $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$. If P is any point on L_2, then show that the distance from P to L_1 is equal to $|C_2 - C_1|/\sqrt{A^2 + B^2}$.