

**INDIAN INSTITUTE OF TECHNOLOGY BOMBAY**

**Department of Mathematics**

Spring Semester 2005

Common Quiz II in MA 104: Mathematics II

**Duration:** 8.30 a.m.-9.10 a.m.

**Max. Marks:** 10

**Date:** 31 March, 2005

**Weightage:** 10 %

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**Instructions**

- (1) Answers are to be written in the 8-page answerbook provided to you. No more answerbooks will be provided.
- (2) Number all the pages of the answerbook before the quiz begins.
- (3) Prepare an index on the top page as per the following format:

<b>Question Number</b>	1	2	3	4
<b>Answer on page No.</b>				

- (4) Write your Roll Number, Name, Division and Batch Number.
  - (5) There is a penalty of two marks if the instructions are not followed.
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**Question 1:** Find the  $LU$ -decomposition of the matrix

$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}.$$

[2 marks]

**Question 2:** Find the dimension of the kernel of the linear map  $F : \mathbb{R}^4 \longrightarrow \mathbb{R}^3$  given by

$$F \left( \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} x + w \\ y + z + w \\ 0 \end{bmatrix}.$$

[2 marks]

**Question 3:** Let  $V$  denote the vector space of all polynomials with real coefficients having degree less than or equal to 5. Put  $\mathcal{B} = \{1, t, t^2, t^3, t^4, t^5\}$ . Let  $D : V \longrightarrow V$  be the linear map

$$D(f(t)) = \frac{df}{dt}.$$

- (a) Find the matrix  $M_{\mathcal{B}}^{\mathcal{B}}(D)$  of  $D$  with respect to the basis  $\mathcal{B}$ .
- (b) Show that the matrix  $M_{\mathcal{C}}^{\mathcal{C}}(D)$  of  $D$  with respect to any basis  $\mathcal{C}$  of  $V$  satisfies  $M_{\mathcal{C}}^{\mathcal{C}}(D)^6 = O$ .

[3 marks]

**Question 4:** Let  $A$  and  $B$  be  $n \times n$  matrices with  $AB = O$ .

- (a) Show that any vector  $u$  in the column space of  $B$  satisfies the equation  $Au = O$ .
- (b) Show that  $\text{rank}(A) + \text{rank}(B) \leq n$ .

[3 marks]

**PAPER ENDS**