MA 523 HW1

Problems stated in the class + the following.

8 **Exercises**

(i) Find integers x, y such that $95x+432y=1$.

(ii) Find integers x, y, z such that $35x+55y+77z=1$.

(iii) Prove that $1+\frac{1}{2}+\cdots+1/n$ is not an integer for $n>1$.

(iv) Prove that
$(\{a, b\}, \{b, c\}, \{c, a\}) = \{(a, b), (b, c), (c, a)\}$.

(v) Prove that if $g_1, g_2, \ldots$ are integers >1 then every natural number can be expressed uniquely in the form $a_0+a_1g_1+a_2g_1g_2+\cdots+a_kg_1\cdots g_k$, where the a_j are integers satisfying $0\leq a_j<g_{j+1}$.

(vi) Show that there exist infinitely many primes of the form $4n+3$.

(vii) Show that, if 2^n+1 is a prime then it is in fact a Fermat prime.

(viii) Show that, if $m>n$, then $2^{2^n}+1$ divides $2^{2^m}-1$ and so $(2^{2^m}+1, 2^{2^n}+1)=1$.

(ix) Deduce that $p_{n+1}\leq 2^{2^n}+1$, whence $\pi(x)\geq \log\log x$ for $x\geq 2$.