

Foreword

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This special issue of the *Journal of Algebra and its Applications* is dedicated to the memory of one of the founding editors of this journal and our *guru* Professor Shreeram Shankar Abhyankar.

Abhyankar had a unique perception of mathematics in general and algebraic geometry in particular. For him, mathematics was not just a tool to solve practical problems of the world, nor was it a purely deductive exercise in manipulating axioms and definitions. He could see mathematics as an intrinsically beautiful discipline with numerous interesting theorems to be discovered to enhance our understanding of some higher reality. Mathematics would reveal its secrets after hard work and then reveal more interesting vistas to explore. In the discovery stage, he was motivated by fertile imagination, a firm belief that a true statement had internal beauty and symmetry and something that needed to be pursued with total concentration and intense meditation. However, once discovered, he preferred that written communication of Mathematics follows a strict discipline. His writings were much influenced by that of C. Chevalley, and over the years he developed a unique style of presentation. His research papers, often considerably long (and some taking the form of monographs), were usually hard to penetrate, but left no one in doubt of their veracity. He insisted on spelling out all little details while writing papers and believed that proofs must be written in such a way that the conclusion must follow without invoking reader's imagination or intuition. He also spent enormous efforts in creating an internal symmetry and structure in such a way that related arguments had a similar structure. He considered this as a pleasing activity as well as a testimony to the accuracy of the deductions.

A number of articles on the life and work of Abhyankar have appeared since his passing away in November 2012. We refer to [1] and [2], and the references therein. This volume is mainly a tribute by way of original research articles or expository surveys, and deal with some of the favorite topics of Abhyankar. We give below a brief description of the fourteen papers included in this issue.

Many of the papers belong to the area of affine algebraic geometry and are related to a pathbreaking result of Abhyankar (and Moh), called the epimorphism theorem. These include the papers by **Makar-Limanov** and by **Palka** giving new proofs of the epimorphism theorem; the former by techniques following the original approach together with some new ideas, while the latter bringing advances in birational geometry of surfaces to establish the theorem. Palka's paper also deals with a related theorem of Lin and Zaidenberg which is a fitting successor to the epimorphism theorem and was a favorite of Abhyankar. The contrast of the two papers is noteworthy. The article by **Dutta and Gupta** gives a panoramic view of problems that are shoots from the seeds of the Abhyankar–Moh theorem. These include generalized epimorphism problems, cancellation problems, and also include some results and questions on affine fibrations and linearization problems. The cancellation problem for the polynomial ring $k[x, y]$, where k is an arbitrary field of positive characteristic, is discussed in the paper of **Bhatwadekar and Gupta**.

The above four papers are followed by a set of three papers on topics that are not far from affine geometry. The paper by **Cassou-Noguès and Daigle** discusses the problem of field generators in two variables, also cultivated by Abhyankar with his student Jan and further developed by Russell and others. It also discusses birational geometry of the affine plane. The concept of locally nilpotent derivations has played an increasingly significant role in problems on automorphisms and epimorphisms of polynomial rings. The paper by **Kuroda** gives a proof of a conjecture of Van den Essen showing why locally nilpotent derivations are essential. Finally, the paper of **Artal Bartolo, Luengo, and Melle-Hernández** discusses the theory of dicritical divisors and curvettes which fascinated Abhyankar during last few years of his mortal life. Indeed his last published paper, completed by and jointly written with Artal Bartolo, discussed the analytic theory of dicritical divisors and curvettes.

From affine geometry, we move to algebraic geometry with a somewhat different flavor. The paper by **Cutkosky** is concerned with a certain problem of Teissier, and it provides a vast generalization of the construction of positive intersection products due to Boucksom, Favre, and Jonsson in the case of a complete algebraic variety over an algebraically closed field. The paper by **Orevkov** goes into classical plane geometry and describes explicit parametric families of plane sextic curves with double points and maximal Milnor number. The paper by **Ghorpade and Krattenthaler** uses combinatorial techniques to show how the a -invariant of coordinate rings of a large class of ladder determinantal varieties can be computed. It may be worthwhile to note that the ladder determinantal varieties were introduced by Abhyankar since they arose in his study of singularities of Schubert varieties in

flag manifolds. These turn out to be natural and nice generalizations of classical determinantal varieties and have been of much interest.

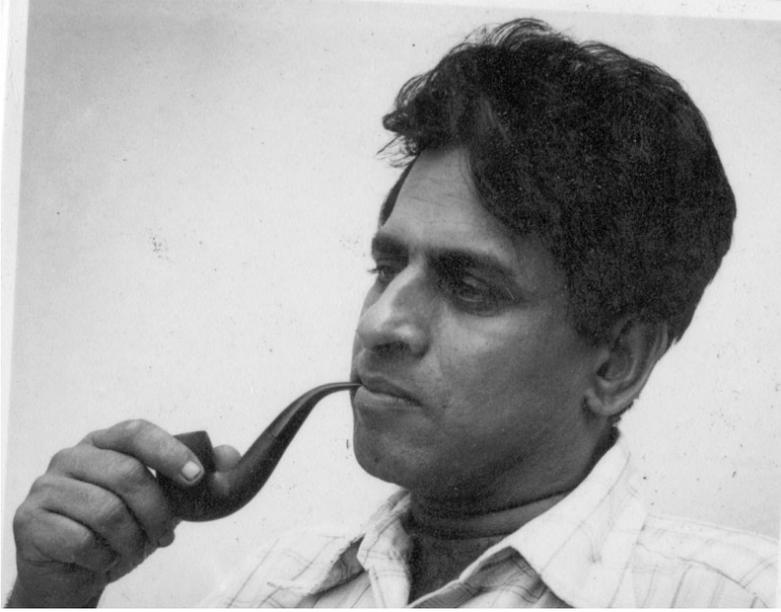
During the late 1980's, Abhyankar was motivated by questions of Serre to renew his interest in coverings of algebraic curves and algebraic fundamental groups. He then worked extensively on Galois theory and group theory for over a decade and constructed many “nice” equations with prescribed Galois groups over algebraic function fields of one variable in characteristic p . The paper by **Mulay** gives a glimpse of Galois Theory that Abhyankar delighted in by proving that an arbitrary group can be realized as the automorphism group of a colored power-set graph.

While Abhyankar usually concentrated on topics that he considered fundamental, there were some nearby topics that also fascinated him. Our last group of papers has three such “friendly topics”. The paper by **Bermejo, García-Marco and Reyes** goes into many of Abhyankar's favorite themes. It discusses toric ideals of undirected graphs and gives algorithms to check if they are complete intersections. It also describes the structure of graphs with such complete intersection ideals. The paper by **Loomis** delves into elementary number theory and gives new results about “friendly” and “solitary” numbers. Finally, the paper by **Lara Rodríguez and Thakur** discusses multiplicative relations between coefficients of logarithmic derivatives of certain formal power series with coefficients in an extension of a finite field. These are then applied to establish new identities between important quantities of function field arithmetic. Thus it is seen that the last paper and in fact several other papers in this issue bring alive the famous exhortation of Abhyankar: *Polynomials and power series, may they forever rule the world!*

We would like to conclude by thanking the *Journal of Algebra and its Applications* and especially its executive editor Professor S. K. Jain for conceiving the idea of this special issue and for inviting us to be its guest editors. We are also grateful to all the authors for their valuable contributions and to the anonymous referees for an excellent and meticulous job of reviewing the papers. The staff of the journal, especially the production editor Rok Ting Tan and the two executive editors S. K. Jain and S. R. López-Permouth, have always been very helpful, and we would like to express our gratitude to them for their kind cooperation.

References

- [1] S. R. Ghorpade, Remembering Shreeram S. Abhyankar, With appendices by A. Sathaye, B. Singh, S. D. Cutkosky and R. V. Gurjar, *Asia Pac. Math. Newsl.* **3**(1) (2013) 22–30.
- [2] S. Mulay and A. Sathaye (Coordinating Eds.), Shreeram Abhyankar (July 22, 1930–November 2, 2012), With contributions by Y. Abhyankar, C. Bajaj, S. D. Cutkosky, D. Harbater, W. Heinzer and D. Shannon, S. Mulay, D. Mumford, A. Sathaye and D. Thakur, *Notices Amer. Math. Soc.* **61**(10) (2014) 1196–1216.



Shreeram S. Abhyankar (1930-2012)