QE in Algebra: Syllabus

Field extensions: Algebraic extensions, algebraic closure, normal extensions, separable extensions, finite fields, inseparable extensions. (Dummit-Foote: Chapter 13, Lang: Chapter V)

Galois theory: Galois extensions, linear independence of characters, norm, trace and discriminants, Hilbert theorem 90, cyclic extensions, solvable and radical extensions, Kummer theory, algebraic independence of homomorphisms, the normal basis theorem. (Dummit-Foote: Chapter 14, Lang: Chapter VI)

Ring extensions: Integral extensions, integral Galois extensions, prime ideals in integral ring extensions, decomposition and inertia groups, ramification index and residue class degree, Frobenius map. (Dummit-Foote: Chapter 16, Lang: Chapter VII)

Modules over a PID and its applications. (Dummit-Foote: Chapter 12, Lang: Chapter III.7)

Noetherian modules and rings: Primary decomposition, Nakayama's lemma, filtered and graded modules, the Hilbert polynomial, Artinian modules and rings. (Dummit-Foote: Chapter 15, Lang: Chapter X)

Semisimple and simple rings: Semisimple modules, Jacobson density theorem, semisimple and simple rings, Wedderburn-Artin structure theorems, Jacobson radical. (Dummit-Foote: Chapter 18, Lang: Chapter XVII)

Representations of finite groups: Basic definitions, characters, class functions, orthogonality relations, induced representations and induced characters, Frobenius reciprocity, decomposition of the regular representation. (Dummit-Foote: Chapter 18, Lang: Chapter XVIII)

References:

1. David S. Dummit, Richard M. Foote: *Abstract Algebra*, second edition, John Wiley and sons, Inc., 2005.
2. Nathan Jacobson: *Basic algebra*, Vol. I-II, second edition, Dover publications, Inc., 2009.
3. Serge Lang: *Algebra*, GTM 211, third edition, Springer-Verlag, 2004.

Syllabus for Qualifying Examination in ANALYSIS

Measure Theory

Measurable sets; Lebesgue Measure and its properties. Measurable functions and their properties; Integration and Convergence theorems.

Introduction to L^p spaces, Riesz-Fischer theorem; Riesz Representation theorem for L^2 spaces.

Absolute continuity of measures, Radon-Nikodym theorem. Duals of L^p spaces.

Product measure spaces, Fubini's theorem.

Fundamental Theorem of Calculus for Lebesgue Integrals.

Functional Analysis

Hahn-Banach Extension and Separation Theorems. Banach spaces. Dual spaces and transposes.

Uniform Boundedness Principle and its applications.

Closed Graph Theorem, Open Mapping Theorem and their applications.
Spectrum of a bounded operator. Spectral theorem for compact self-adjoint operators.

Hilbert spaces. Orthonormal basis.
Projection theorem and Riesz Representation Theorem for Hilbert spaces.

Complex Analysis

Analytic Functions, Harmonic functions, Cauchy-Goursat Theorem, Conformal mappings.

Taylor and Laurent series, Isolated singularities and residues, Zeroes and poles.

Maximum Modulus Principle, Argument Principle, Rouche's theorem.

Liouvilles Theorem, Morera's Theorem.

REFERENCES

1. H. L. Royden, Real Analysis, 3rd Ed., Prentice Hall of India, 1988.

2. B. V. Limaye, Functional Analysis, 3rd Ed., New Age International Publishers, 2014.

3. J. B. Conway, Functions of one complex variable, 2nd Edition, Narosa, New Delhi, 1978.

Suggested Chapters/Sections from the reference books

Measure Theory

1. H. L. Royden, Real Analysis, 3rd Ed., Prentice Hall of India, 1988.

Chapter	Sections
3	1, 2, 3, 5, 6
4	1, 2, 3, 4
5	1, 2, 3, 4
6	1, 2, 3, 5
11	1, 2, 3, 4, 5, 6
12	1, 2, 4

Functional Analysis

2. B. V. Limaye, Functional Analysis, 3rd Ed., New Age International Publishers, 2014.

Chapter	Sections
II	5, 6, 7, 8
III	9, 10, 11, 12
IV	13, 14
VI	21, 22, 24
VII	28: Lemma 28.4, Theorem 28.5

Complex Analysis

3. J. B. Conway, Functions of one complex variable, 2nd Edition, Narosa, New Delhi, 1978.

Chapter	Sections
III	1, 2, 3
IV	1, 2, 3, 4, 5
V	1, 2, 3
VI	1
X	1, 2

Qualifiers in Combinatorics and Theoretical Computer Science

References:
Extremal Combinatorics with Applications in Computer Science - Jukna
Chapters 4,5,6,7,8,10,13,16.

Enumerative Combinatorics - Stanley vols 1 and 2 Chapters 1,2,3,5

Extremal Combinatorics: Pigeonhole principle (Jukna Chap 4), Matchings and SDRs (Jukna Chap 5), Sunflower Lemmas (Jukna Chap 6), Intersecting Families (Jukna Chap 7), Chains and antichains (Jukna Chap 8), Density theorems (Jukna Chap 10), Linear Algebra method (Jukna Chap 13), Polynomial method (Jukna Chap 16).

Enumerative Combinatorics: Ordinary Generating functions (Stanley v1 - Chaps 1,2)
Exponential Generating Functions (Stanley v2 - Chap 5),
Mobius Inversion on Posets (Stanley v1 - Chap 3),
Trees, Composition of generating functions and Exponential Formula (Stanley v2 - Chap 5).

References:
Extremal Combinatorics with Applications in Computer Science - Jukna

Enumerative Combinatorics - Stanley vols 1 and 2 (Chapters 1,2,3 and 5)

Qualifier Examination Syllabus
for Differential Equations

Initial Value Problems for Ordinary Differential Equations: Existence and uniqueness of solutions, continuation of solutions and maximal interval of existence, continuous dependence. ***(Sections 1 to 7 in Chapter 1 of [1], and Sections 2.1 to 2.4 of [2])***

First Order Autonomous Systems: Solution of linear systems, phase space analysis, critical points, proper and improper nodes, spiral points and saddle points**.** ***(Chapter 1 of [2])***

Linearization, Lyapunov methods. ***(Sections 2.5, 2.6, and 2.9 of [2])***

Nonlinear First-Order Scalar Partial Differential Equations: Method of characteristics, scalar conservation laws: shocks and entropy condition, weak solutions and R-H condition, and long time behavior**.** ***(Chapter 3 of [3])***

Linear Elliptic Partial Differential Equations: Weak solutions, Lax-Milgram theorem and its applications. Regularity of solutions to Dirichlet problem on a bounded domain, Poissons' equation in full space. Maximum principles, eigenvalue problems. ***(Chapter 3 of [4] and Chapter 6 of [3])***

Linear Evolution Partial Differential Equations: Existence of solutions, regularity, and maximum principles for second order parabolic equations, Existence of solutions, regularity, propagation of disturbances for second order hyperbolic equations. ***(Sections 7.1 and 7.2 of [3])***

References:

1. E.A. Coddington and N. Levinson, Theory of ordinary differential equations, Tata McGraw Hill, 1987
2. L. Perko, Differential Equations andDynamical Systems, 3rd edition, Springer Verlag, 2001.
3. L C. Evans, Partial Differential Equations, 2nd Edition, American Mathematical Society, 2010.
4. S. Kesavan, Topics in Functional Analysis and Applications, John Wiley and Sons, 1989.

Syllabus – Probability Qualifier

Part- I : Probability

Probability Space, Probability mesaures, construction of Lebesque measure, Extension theorem [Chapater 2]
Random variables and Random vectors, distributions, multi distributions, independence
Expectation of a random variable, Change of variable theorem, Sequence of random variables, convegence theorems. [Chapter 3]

Convergence almost surely, in probability, in law, convergence in distribution, limit of events, Borel -Cantelli lemma [Chapter 4, 4.1-4.4]

Moment genertaing functions, Characteristic functions, Uniqueness theorem,
Inversion theorem, continuity theorem [Chapter 6]

Weak law of large numbers, strong law of large numbers, central limit theorem,
[Chpater 5, Chpter 7: 7.1-7.3]

Radon Nikodym theorem, Condition expectation definition, existence and
its properties. [Chpater 9: 9.1]

Reference: K L Chung, A course in probability theory, 3rd edition, Academic Press, San Dieago, 2001.

Part -II : Stochastic Processes

Discrete time Markov chains, Markov property, transition kernels, invariant distributions, recurence, transients, ergodic behavior of irreducible chains.
[Chapter 2, 1-7, Chapter 3, 1-5]

Homogeneous and non-homogeneous Poisson processes, [Chapter 4, 1-3]

Martingales, sub and super martingales, stopping times. [Chapter 6, 1-2]

Karlin, S. And Taylor, H.M., A first course in stochastic processes, 2nd Edition,
Academic Press, NewYork, 1975.

Syllabus Statistics Qualifier (Applicable to Students admitted to the PhD program in the Department of Mathematics, IITB, from Spring 2022 onwards)

Inference:

1) Parametric models, exponential and location-scale family, Sufficiency, Minimal Sufficiency, Complete Statistic, Decision Rule, Loss Function and Risk, Point estimators, consistency, asymptotic bias, variance and MSE, asymptotic inference.[Chapter 2 of Shao 2003]

2) UMVUE, U-statistics, Asymptotic Unbiased estimator, V-statistics [Chapter 3 of Shao 2003]

3) Bayes Decision and Bayes estimators, Invariance, Minimaxity and admissibility,

MLE and efficient estimation method. [Chapter 4 of Shao 2003]

4) The NP Lemma, monotone likelihood ratio, UMP test for one sided and two sided

hypothesis, UMP Unbiased test, UMP invariant test, likelihood ratio test, chi-squared test, Sign, permutation and rank test, Kolmogorov- Smirnov and Cramer-von Mises test and asymptotic test [Chapter 6 of Shao 2003]

Main Text: Mathematical Statistics, Jun Shao, 2nd Ed.,Springer, 2003.
Additional Texts:

1) Theoretical Statistics D.R. Cox, D.V. Hinkley CRC Press, 1974.

2) Theory of Statistical Inference, E. L. Lehmann, Wiley, 1983.

3) Testing Statistical Hypotheses, E. L. Lehmann, Wiley, 1986.

4) Theory of Statistics, Mark J. Schervish, Springer, 1995.

Regression and Statistical Modelling:

1) Full rank model (Chapters 3 and 4 of Searle 1971)
2) One-way classification model (Section 6.2 of Searle 1971)
3) Two-way Crossed Classification model (Chapter 7: Sections 7.1,7.2 of Searle 1971)

Main Text: Linear Models by S.R. Searle (1971) Wiley & Sons

Additional Texts: 1) Linear Model Methodology by A. I. Khuri (2009) CRC Press

<u>Existing Syllabus for Statistics Qualifier: (Applicable to Students admitted to the PhD program in the Department of Mathematics, IITB, prior to Spring 2022)</u>

Inference:
Chapter 6: Principles of data reduction Chapter 7: Point Estimation
Chapter 8: Hypothesis testing
Chapter 9: Interval estimation
Chapter 10: Asymptotic evaluations

Main Text: Statistical Inference by Casella and Berger

Regression

Chapter 1 Introduction
Chapter 2 Multiple regression
Chapter 3 Tests and confidence regions Chapter 4 Indicator Variable
Chapter 5: The normality Assumption
Chapter 6: Unequal variances
Chapter 7; Correlated errors
Chapter 8: Outliers and influential observations Chapter 10 Multicollinearity
Chapter 11 Variable selection

Main Text: Regression Analysis by Sen and Srivastava

Topic: Geometry and Topology

Differentiable manifolds, differentiable functions, tangent spaces, inverse function theorem, local immersion and local submersion theorems, vector fields, differential forms, de Rham cohomology, orientation, integration on manifolds, Stokes theorem. Statement and applications (without proof) of Poincare duality for differential forms.

References for the above: Chapters 1-4 and first three sections of Chapter5 of Bott and Tu.

Fundamental groups, Van-Kampen theorem, examples of fundamental groups of projective spaces, circle, tori, surfaces. Covering spaces, deck transformations, lifting theorem, universal cover, G- coverings, Galois correspondence for covering spaces.

References for the above: Chapters 11 to 14 in Fulton

Cones, mapping cylinder, suspensions. Singular homology (homotopy invariance, excision, long exact sequence of pairs, Mayer-Vietoris sequence), examples of computing singular homology. Statement and applications (without proof) of Kunneth formula for singular homology.

References for the above: (a)Section 1 of Chapter 2 in Hatcher (for everything except Kunneth and Mayer-Vietoris sequence), or (b) Sections 8 to 17 in Greenberg and Harper, and section 29 for Kunneth formula.

Books:

1. Bredon, Glen E. : Topology and geometry. Corrected third printing of the 1993 original. Graduate Texts in Mathematics, 139. Springer-Verlag, New York, 1997. xiv+557 pp. ISBN: 0-387-97926-3 55-01 (54-01 57-01)

2. Fulton, William : Algebraic topology. A first course. Graduate Texts in Mathematics, 153. Springer-Verlag, New York, 1995. xviii+430 pp. ISBN:0-387-94326-9; 0-387-94327-7 55-01 (30-01 57-01)

3. Vick, James W. : Homology theory. An introduction to algebraic topology. Second edition. Graduate Texts in Mathematics, 145. Springer-Verlag, New York, 1994. xiv+242 pp. ISBN: 0-387-94126-6 55-01
(57-01)

4. Hatcher, Allen : Algebraic topology. Cambridge University Press, Cambridge, 2002. xii+544 pp. ISBN: 0-521-79160-X; 0-521-79540-0 55-01 (55-00)

5. Bott, Raoul; Tu, Loring W. : Differential forms in algebraic topology. Graduate Texts in Mathematics, 82. *Springer-Verlag, New York-Berlin,* 1982. xiv+331 pp. ISBN: 0-387-90613-4

6. Greenberg, Marvin J.; Harper, John R. : A first course. Mathematics Lecture Note Series, 58. *Benjamin/Cummings Publishing Co., Inc., Advanced Book Program, Reading, Mass.,* 1981. xi+311 pp. (loose errata). ISBN:0-8053-3558-7; 0-8053-3557-9