MATHEMATICS OLYMPIAD

Mathematics Association IIT Bombay

Level: I

Name: _____ **Roll Number:** _____

INSTRUCTIONS TO CANDIDATES

- **Do not open this booklet until instructed by the invigilators.**
- **Time Allowed:** 3 Hours. **Maximum Marks:** 100. **Date:** 8th February 2026
- This question paper booklet consists of **8** printed pages. Please check that this booklet is complete and does not have any missing or unprinted pages.
- Questions must be answered on the **Optical Mark Recognition sheet (OMR)** by darkening the appropriate squares using a **blue/black ball pen**.
- Write your registration number, name, and the Question Paper code in the space provided on the OMR sheet.
- This question paper contains **25** questions divided into two sections.
- **Section A (Q1–Q20):** Contains 20 Multiple Choice Questions (MCQ).
  - **Marking Scheme: +4** for correct answer, **-1** for incorrect answer, **0** if not attempted.
- **Section B (Q21–Q25):** Contains 5 Integer Type Questions. The answer is a **3-digit integer** between 000 and 999.
  - Candidates must darken three squares, one for each digit (e.g., 9 as 009, 10 as 010).
  - **Marking Scheme: +4** for correct answer, **0** for incorrect answer or unattempted.
- You won't be allowed to leave the examination hall during the exam.
- Space for rough work is provided at the bottom of each page.

Space for Rough Work

SECTION A: Multiple Choice Questions (20 Questions)

Select the correct option. Each question carries 4 marks. -1 mark for incorrect answers.

Q1. Find the 20th digit after the decimal in $(5+\sqrt{26})^{20}$.

(A) 1 (B) 8 (C) 9 (D) 0

Q2. Let $f(x)$ be a polynomial such that $f(x) \cdot f(1/x) = f(x) + f(1/x)$ and $f(3) = 28$. Find $f(4)$.

(A) 63 (B) 65 (C) 17 (D) 82

Q3. A Super GP is a sequence such that the natural logarithms of its terms form a geometric progression. Let S denote the Super GP with super ratio x and first term y, such that $x^2 - 4y + y^2 + 29 - 10x = 0$ ($x, y \in \mathbb{R}$). Let S_n denote the sum of the first n terms of this Super GP. Find the remainder when S_{100} is divided by 17.

(A) 1 (B) 0 (C) 8 (D) 16

Q4. Let $f(x)$ be a function defined for non-zero real numbers satisfying $2f(x) + f(1/x) = 3x$. Find the value of $f(2)$.

(A) 2 (B) 3.5 (C) 4.5 (D) 7

Q5. Let $k > 2$ be a natural number such that regular k-sided polygons can tile the plane. (Tiling the plane means that infinitely many copies of the polygon can cover the plane completely without gaps or overlaps). Find the sum of all possible values of k.

(A) 11 (B) 12 (C) 13 (D) 15

Q6. In $\triangle ABC$, a point D lies on side BC such that AD is the internal angle bisector of $\angle A$. It is given that $\angle B = 2\angle C$ and $CD = AB$. Determine the measure of $\angle A$.

(A) 36° (B) 60° (C) 72° (D) 108°

Q7. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the top faces is a prime number?

(A) $5/12$ (B) $7/12$ (C) $1/2$ (D) $13/36$

Q8. Let $ABCD$ be a rectangle such that $AB + BC + CD = 20$. Let E be the midpoint of side BC and suppose $AE = 9$. Find the area of rectangle $ABCD$.

(A) 15 (B) 17 (C) 19 (D) 21

Q9. Find the number of zeros at the end of $100!$ when written in base 12.

(A) 48 (B) 97 (C) 24 (D) 50

Q10. Any number can be Sliced into a sequence of numbers (e.g., 365 can be sliced as 3|6|5, 36|5, 3|65, etc.). A number is *slimy* if it can be sliced into a sequence of primes.
Examples: 5 is slimy (5 is prime). 2027 is slimy (sliced as 2, 02 (which is 2), 7).
A number is *sticky* if you obtain a sequence of primes no matter how you slice it.
Example: 53 is sticky, since both (53) and (5, 3) are sequences of primes.
Let S denote the set of all the sticky numbers. Find the cardinality of S.

(A) 9 (B) 12 (C) 15 (D) ∞

Q11. Let ABC be a triangle with fixed sides $AB = 15$ and $AC = 10$. A rhombus $ADEF$ is constructed inside the triangle such that vertex A is common, D lies on AB, F on AC, and E on BC. If the sides of the rhombus are parallel to the sides of the triangle, determine the side length of the rhombus.

(A) 4 (B) 5 (C) 6 (D) 7

Q12. Consider the arithmetic progression $A_n = 4n - 3$. Let S be the set consisting of the first 20 terms of this progression. Five numbers are chosen at random from S. Find the maximum possible value of the absolute difference between the median and the mean of the chosen numbers.

(A) 20 (B) 22 (C) 24 (D) 26

Q13. In $\triangle ABC$, point D lies on BC such that $BD : DC = 1 : 2$ and point E lies on AC such that $AE : EC = 3 : 2$. Segments AD and BE intersect at P. If the area of $\triangle ABC$ is 110, find the area of $\triangle APB$.

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(A) 25 (B) 28 (C) 30 (D) 32

Q14. Let ABC be a triangle with $\angle A = \theta$. Extend BC beyond C to D. Let the internal angle bisectors of $\angle ABC$ and $\angle ACD$ intersect at A_1. Applying the same construction to $\triangle A_1BC$ produces A_2, and so on. If $\angle A_1 + \angle A_2 + \angle A_3 + \angle A_4 + \angle A_5 = 93^\circ$, find θ.

(A) 90 (B) 120 (C) 96 (D) 48

Q15. Fifty real numbers $x_1, x_2, \ldots, x_{50}$ all have the same absolute value. Each x_i is either positive or negative. Let $S = \sum_{1 \le i < j \le 50} x_i x_j$. It is given that S is positive and is as small as possible among all sign choices. Suppose exactly k of the numbers are positive. Find the value of $k^2 + (50 - k)^2$.

(A) 1280 (B) 1282 (C) 1284 (D) 1300

Q16. Find the number of ordered pairs (x, y) of integers such that $x, y \in \{1, 2, \ldots, 20\}$ and $x^2 + y^2 \equiv 0 \pmod 5$.

(A) 120 (B) 144 (C) 150 (D) 100

Q17. Consider two Arithmetic Progressions: $A_1 = \{1, 3, 5, 7, 9, \ldots, 2025\}$ and $A_2 = \{1, 10, 19, 28, 37, \ldots, 2026\}$. Let $A_3 = A_1 \cap A_2$. Find the number of terms in A_3 that are perfect squares.

(A) 1 (B) 10 (C) 5 (D) 4

Q18. Let ABC be a right-angled triangle with $\angle A = 90^\circ$. Given $AB = 10$ and $AC = 12$, let a point D lie on BC such that $BD : DC = m : n$, where m, n are natural numbers. Among all such points D, the segment AD is shortest for a particular ratio $m : n$. Find the minimum possible value of $m^2 + n^2$.

(A) 841 (B) 1025 (C) 1921 (D) 2500

Q19. A rectangular box has dimensions x, y, z. The sum of all its 12 edges is 60, and its total surface area is 144. Let V_{max} and V_{min} be the maximum and minimum possible volumes of such a box. Find the value of $V_{\text{max}} - V_{\text{min}}$.

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(A) 2 (B) 4 (C) 6 (D) 8

Q20. A rectangle has area 36. Both its length and breadth are composite natural numbers. Find the sum of all possible perimeters of the rectangle.

(A) 48 (B) 50 (C) 52 (D) 60

SECTION B: Integer Type Questions (5 Questions)

The answer to each question is a 3-digit integer. No negative marking.

Q21. A rook is placed on a 2026×2026 chessboard. A rook moves any number of squares horizontally or vertically. Let N be the number of squares the rook can reach in exactly one legal move. Find the sum of the digits of the integer nearest to the expected value of N.

Q22. Let x, y, z be positive real numbers such that $x + y + z = 1$. Find the minimum value of $\frac{1}{x} + \frac{4}{y} + \frac{9}{z}$.

Q23. Let $S_n = 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n!$. Find the remainder when S_{100} is divided by 101.

Q24. A triathlon is an endurance multisport race consisting of swimming 4 km, cycling 180 km, and running 42 km. Triathletes compete for the fastest overall completion time, racing each segment sequentially with the time transitioning between the disciplines included. Two seasoned athletes decided to compete in this event, one named Bed and other named Rock. Since the athletes are seasoned, they race each segment with a constant rate without fatigue or slowing down. Their rates are as follows:
Bed: Swimming-1.6 km/h, Cycling-30 km/h, Running-7km/h.
Rock: Swimming-2.5 km/h, Cycling-24 km/h, Running-10.5km/h.
Bed decides to first swim, then cycle, then run. Rock goes in the other direction, first running, then cycling to finish off with swimming. Since the track is the same for all triathletes, find the remaining time in minutes that Rock will take to complete the race when they meet on the track midway going past each other?

Q25. Find the largest integer n such that $n + 10$ divides $n^3 + 100$.

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