MATHEMATICS OLYMPIAD

Mathematics Association IIT Bombay

Level: II

Name: _____ **Roll Number:** _____

INSTRUCTIONS TO CANDIDATES

- **Do not open this booklet until instructed by the invigilators.**
- **Time Allowed:** 3 Hours. **Maximum Marks:** 100. **Date:** 8th February 2026
- This question paper booklet consists of **8** printed pages. Please check that this booklet is complete and does not have any missing or unprinted pages.
- Questions must be answered on the **Optical Mark Recognition sheet (OMR)** by darkening the appropriate squares using a **blue/black ball pen**.
- Write your registration number, name, and the Question Paper code in the space provided on the OMR sheet.
- This question paper contains **25** questions divided into two sections.
- **Section A (Q1–Q20):** Contains 20 Multiple Choice Questions (MCQ).
  - **Marking Scheme: +4** for correct answer, **-1** for incorrect answer, **0** if not attempted.
- **Section B (Q21–Q25):** Contains 5 Integer Type Questions. The answer is a **3-digit integer** between 000 and 999.
  - Candidates must darken three squares, one for each digit (e.g., 9 as 009, 10 as 010).
  - **Marking Scheme: +4** for correct answer, **0** for incorrect answer or unattempted.
- You won't be allowed to leave the examination hall during the exam.
- Space for rough work is provided at the bottom of each page.

Space for Rough Work

SECTION A: Multiple Choice Questions (20 Questions)

Select the correct option. Each question carries 4 marks. -1 mark for incorrect answers.

Q1. Let $y(x)$ be the solution to the differential equation:

$$\frac{dy}{dx} + \frac{2026x^{2025}}{1+x^{2026}}y = \frac{2026x^{2025}}{1+x^{2026}}$$

with the initial condition $y(0) = 0$. Evaluate $\lim_{x\to\infty} y(x)$.

(A) 0 (B) 1 (C) 2026 (D) ∞

Q2. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuously differentiable function that satisfies $f'(t) > f(f(t))$ for all $t \in \mathbb{R}$. Then which of the following is **NOT ALWAYS** true?

(A) $f(f(f(0))) \leq 0$

(B) $f(1) < 1$

(C) $f(f(2)) < f(2)$

(D) if $f(3) \leq 0$ and $f(f(3)) > 0$, then $f(s) > f(3)\ \forall s > 0$

Q3. Let $\mathbb{P}$ denote the set of prime natural numbers. Two distinct elements are chosen from $\mathbb{P}$ (call them P_1 and P_2). If the number of non-negative integral solutions of $P_1x + P_2y = 1000$ turns out to be 13, find the value of P_1P_2.

(A) 77 (B) 91 (C) 143 (D) 221

Q4. In a triangle ABC, $AB = AC$. A point D is on AC such that $BC \cdot CD = BD^2$. If $\angle ABD = \angle DBC$, then $\angle A$ equals:

(A) 36° (B) 45° (C) 72° (D) 30°

Q5. Find $\frac{I_1}{I_2}$ where $I_1 = \int_1^\infty \frac{dt}{\sqrt{t^3-t}}$ and $I_2 = \int_0^1 \frac{dt}{\sqrt{1-t^4}}$.

(A) 1 (B) 2 (C) 4 (D) 1/2

Q6. Let $S(\neq \mathbb{R}^3)$ be a set of points in $\mathbb{R}^3$ such that for any two points in S, the entire line joining them lies completely in S. Suppose S passes through $(1, 2, 1)$, $(3, 0, 3)$, and $(-1, 2, 3)$. Let C denote the locus of all points in space at a distance 5 units from the origin. Let $P = S \cap C$. Let Q be the projection of the curve P onto the xy-plane. Find the area enclosed by Q.

(A) $\frac{19\pi}{\sqrt{6}}$ (B) 19 (C) $\frac{25\pi}{\sqrt{6}}$ (D) $6\sqrt{19}\pi$

Q7. The polynomial $P(z) = (z + i)(z + 2i)(z + 3i) + 10$ has three roots in the complex plane. Let these roots be the vertices of a triangle T. What is the area of T?

(A) 6 (B) 8 (C) 10 (D) $6\sqrt{3}$

Q8. A narrow corridor consists of 10 consecutive segments, each of length 1 unit, and has a fixed width of 2 units. Security tiles of size 1×2 (which may be placed either vertically or horizontally) are to be laid so that the entire corridor is covered exactly. In how many distinct ways can the corridor be fully tiled?

(A) 55 (B) 89 (C) 144 (D) 233

Q9. Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = x^5 + 2x^3 + 4x + 1$. Let f be a continuous function such that $f(g(x)) = x$ for all real x. Evaluate $I = \int_1^8 f(x)dx$.

(A) 3/2 (B) 13/3 (C) 31/6 (D) 5

Q10. Two circles of radius 4 and 6 respectively have their centers separated by 12 cm. M is a point on the first circle and N is a point on the second circle. Find the area of the locus of the midpoint of M and N as (M, N) vary on the circles respectively.

(A) 16π (B) 25 (C) 36 (D) 24π

Q11. C_1 and C_2 are concentric circles of radius 4 and 6 respectively. P is a fixed point on C_1. From P, chords are drawn to C_2 and at the end points of these chords, tangents are drawn. These tangents meet at R. Find the shortest distance of the locus of R from the centre of C_1.

(A) 4 (B) 6 (C) 9 (D) 16

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Q12. Find the number of ways to place at least one knight on a 2×8 board such that no two knights attack each other. (Note: Two knights attack each other if they are in different rows and exactly two columns apart.)

(A) 1025 (B) 2401 (C) 3200 (D) 4095

Q13. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the top faces is a prime number?

(A) 5/12 (B) 7/12 (C) 1/2 (D) 13/36

Q14. Let $f(x)$ be a real polynomial satisfying $f(x) + f(1/x) = f(x)f(1/x)$ for all $x \neq 0$. Given $f(2) = 5$, evaluate the definite integral $I = \int_0^{\pi} \frac{x \sin x}{f(\cos x)} dx$. The value of $\frac{48I}{\pi^2}$ is:

(A) 10 (B) 12 (C) 14 (D) 16

Q15. Consider the arithmetic progression defined by $A_n = 4n - 3$. Let S be the set of the first 20 terms. Five numbers are chosen at random from S. Find the maximum possible value of the absolute difference between the median and the mean of the chosen numbers.

(A) 20 (B) 24 (C) 28 (D) 32

Q16. How many ordered pairs of prime numbers (p, q) satisfy the condition that $p^2 + 3pq + q^2$ is a perfect square?

(A) 0 (B) 1 (C) 2 (D) 4

Q17. Find the 20th digit after the decimal in $(5 + \sqrt{26})^{20}$.

(A) 1 (B) 8 (C) 9 (D) 0

Q18. Technoblade and Ramanujan are close friends. Technoblade has to drink each of the 6 indistinguishable potions (A, B, C, D, E, F) at least once to beat a demon. Ramanujan calculates that the expected number of potions Technoblade drinks until he has drunk each potion at least once is x. The value of x is:

(A) 10.5 (B) 21 (C) 6 (D) 14.7

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Q19. Let x, y, z be real numbers satisfying $x + y + z = 5$ and $x^2 + y^2 + z^2 = 9$. Let z_{max} and z_{min} be the maximum and minimum possible values for z. Find $z_{max} + z_{min}$.

(A) 10/3 (B) 16/3 (C) 11/3 (D) 17/3

Q20. Let $C_1 = \{n \in \mathbb{N} : \log_{10} n < 3\}$. Let C_2 = {$n \in C_1 : n$ is divisible by 4 and the sum of its digits is divisible by 7}. Find the cardinality of the set C_2.

(A) 28 (B) 34 (C) 50 (D) 0

SECTION B: Integer Answer Type Questions (5 Questions)

Write the answer as a 3-digit integer (000–999). No negative marking.

Q21. Consider the polynomial $f(x) := \prod_{k=1}^{5} \left(1 + \frac{kx}{1!} + \cdots + \frac{(kx)^5}{5!}\right)^5$. If the coefficient of x^5 in $f(x)$ is $a_5 = 5^\ell \cdot \frac{a}{b}$ where a and b are coprime numbers not divisible by 5 and ℓ is an integer, then what is the value of ℓ?

Q22. Let $ABCD$ be a non-square rectangle inscribed in a circle, and let E be any arbitrary point on the circle distinct from $ABCD$. Given that the length and width of the rectangle are co-prime integers and the area is 360, find the value of $\sqrt{\frac{AE^2+BE^2+CE^2+DE^2}{2}}$.
Hint: The answer is an exact integer.

Q23. Let $S_n = 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n!$. Find the remainder when S_{100} is divided by 101.

Q24. An ant starts at the origin $(0, 0)$ and wants to reach its home at $(5, 4)$. It can only move one unit at a time to the Right or Up. If there is a drop of poison at $(2, 2)$ that the ant must not touch, how many safe paths are available for its journey?

Q25. Let $f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} \ldots$ for $x \in (-1, 1)$ and $g(x) = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} \ldots$. Let $h(x)$ denote the derivative of $f(x)$. Let $I = \int_0^\pi \frac{g(x)}{h(\cos x)} dx$. Find the value of $\lfloor 8I \rfloor$.

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