Mathematics Olympiad 2025

Mathematics Association IIT Bombay

Name: .....

Reg. No.: M O 2 5 _ _ _ A Level: I

Code: A

Date: Feb 9, 2025

Maximum Marks: 100

Time: 09:30 - 11:30

Read the following instructions carefully before attempting the question paper.

- Do not open this booklet until instructed by the invigilators.
- Questions must be answered on **O**ptical **M**ark **R**ecognition sheet (**OMR**) by darkening the appropriate squares (marked A, B, C, D or $0-9$) using blue/black ball pen.
- Write your registration number, name, and the Question Paper code (**A**, **B**, **C** or **D**) in the space provided on the OMR sheet.
- The question paper consists of three sections: **Section 1**, **Section 2** and **Section 3**.
- **Section 1:** This section consists of **15 objective questions**, each having **exactly one correct answer**. Each correct response will be awarded $+\mathbf{3}$ marks, while each incorrect response will result in a penalty of $-\mathbf{0.75}$ marks. Candidates must darken **exactly one square in the OMR sheet** per question using a black or blue ball pen.
- **Section 2:** This section contains **5 objective questions**, each with **exactly one correct answer**. Each correct response will be awarded $+\mathbf{5}$ marks, and each incorrect response will incur a penalty of $-\mathbf{1.25}$ marks. Candidates must darken **exactly one square in the OMR sheet** per question using a black or blue ball pen.
- **Section 3:** This section comprises **6 numerical answer type questions**, where the answer is an **integer between** 000 **and** 999 **(both inclusive)**. Each correct response will be awarded $+\mathbf{5}$ marks, and no negative marking will be applied. Candidates must darken **three squares, one for each digit, in the OMR sheet**, for every question using a black or blue ball pen. For example, the number 9 should be marked as 009, 10 as 010, and 831 as 831.
- You won't be allowed to leave the examination hall during the exam.

Notation

- For a set A, the notation $x \in A$ means "x is an element of the set A". The notation $|A|$ means the cardinality of set A.
- $A \setminus B$ means the set difference: elements in A but not in B.
- $\mathbb{N}$ = the set of all natural numbers = $\{1, 2, 3, \ldots\}$.
- $\mathbb{Z}$ = the set of all integers = $\{0, -1, 1, -2, 2, \ldots\}$.
- $\mathbb{R}$ = the set of all real numbers.
- For $a, b \in \mathbb{R}$,
  - (i) $[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$.
  - (ii) $[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$.
  - (iii) $(a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$.
  - (iv) $(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$.
  - (v) $a \mid b$ means "a divides b".
  - (vi) $a \equiv b \pmod{k}$ means "k divides $(a - b)$".
- For a natural number n,
$$n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1.$$
- For a sequence $a_1, a_2, \ldots$ and two natural numbers $m < n$,
$$\prod_{i=m}^{n} a_i = a_m \cdot a_{m+1} \cdots a_{n-1} \cdot a_n.$$
- For a real number x, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.
- $[ABC]$ = Area of triangle ABC.
- $\odot(PQR)$ = Circle passing through P, Q, R.
- $f^{(k)}(x)$ means "kth derivative of function f".

Table of Values

Constant	Value
e	2.718
π	3.142
$\ln 2$	0.693
$\ln 3$	1.099
$\ln 5$	1.609
$\ln 7$	1.946

Section 1: Objective Questions

This section consists of **15 objective questions**, each having **exactly one correct answer**. Each correct response will be awarded **+3** marks, while each incorrect response will result in a penalty of **−0.75** marks. Candidates must darken **exactly one square in the OMR sheet** per question using a black or blue ball pen.

Q-1 [q01]⟩ Consider the polynomial

$$g(x) = \prod_{i=1}^{7}(1 + x^{i!} + x^{2i!} + x^{3i!} + \cdots + x^{(i-1)i!} + x^{ii!})$$

Then the coefficent of x^{2025} in the expansion of $g(x)$ is

[A] 0 [B] 1 [C] 2025 [D] 4049

Q-2 [q02]⟩ Let

$$A(n) = \{f : [n] \to [n] \mid f(f(x)) + f(x) = 5 \text{ for all } x \in [n]\}.$$

Find the sum of all primes p such that p divides $|A(p)|$. (Here, $[n]$ is the set $\{1, 2, \ldots, n\}$.)

[A] 16 [B] 18 [C] 22 [D] 29

Q-3 [q03]⟩ Consider the functional equation, $f(x)+3f(1-x) = x^2+2024x+2024$, where f is an integer valued function. How many solutions does the above equation have?

[A] 0 [B] 1 [C] Finitely Many [D] Infinitely Many

Q-4 [q04]⟩ Assume that

$$f(x) = x^3 + ax^2 + 2bx + c \text{ and } g(x) = x^3 + mx^2 + 2nx + p$$

are any two cubic polynomials with a common root d. Consider the following relations

1. Statement I:
$$\left(d + \frac{b-n}{a-m}\right)^2 = \frac{(b-n)^2 - (a-m)(c-p)}{(a-m)^2}$$
2. Statement II:
$$\left(d + \frac{b+n}{a+m}\right)^2 = \frac{(b+n)^2 - (a+m)(c+p)}{(a+m)^2}$$

Then

[A] I is correct, II is correct.
[B] I is correct, II is incorrect.
[C] I is incorrect, II is correct.
[D] I is incorrect, II is incorrect.

Q-5 [q05]⟩ a, b, c, d are integers satisfying $a > b > c > d$ and

$$a + b + c + d = 2004, a^2 - b^2 + c^2 - d^2 = 2004.$$

Then the sum of "smallest possible value of a" and the "number of possible values of a" is

[A] 1001 [B] 1002 [C] 1003 [D] 1004

Q-6 [q06]⟩ Given $x, y \in \mathbb{R}$ such that

$$x + y = 2, x^2 + y^2 = \frac{4}{3}, x^{2025} + y^{2025} = cos(\theta) - sin(\theta),$$

then a possible value of θ is ..?

[A] $\frac{\pi}{2}$ [B] $\frac{3\pi}{4}$ [C] $\frac{5\pi}{2}$ [D] $\frac{5\pi}{4}$

Q-7 [q07]⟩ Let the sum of all natural numbers n for which $\sum_{k=1}^{S(n)} k^3 = n$ be T. Find the sum of the distinct prime divisors of T.
(Here $S(k)$ denotes the sum of digits of k in its decimal representation.)

[A] 537 [B] 1015 [C] 1815 [D] 2037

Q-8 [q08]⟩ Find the number of quadraples (a, b, c, d) of positive integers that satisfy $2(a+b+c+d)-6 = ab + cd$

[A] 8 [B] 9 [C] 10 [D] 11

Q-9 [q09]⟩ Mickey has a quirky game he loves to play. First, he tosses a coin, and if it lands on Heads, he rolls his dice twice. He wins only if he rolls 1 on both attempts. Alternatively if the coin lands on Tails, Mickey rolls his dice only once. If he rolls a 1, Mickey wins. If Mickey doesn't win on that turn, he tosses the coin again and tries all over again. What is the probability that Mickey will win the game in less than or equal to 2 coin tosses?

[A] $\frac{7 \times 167}{2592}$ [B] $\frac{7 \times 167}{5184}$ [C] $\frac{7 \times 137}{2592}$ [D] $\frac{7 \times 137}{5184}$

Q-10 [q10]⟩ Let $ABCD$ be a kite with diagonals of lengths 2 and 3. Moreover, suppose $ABCD$ is both inscribed in a circle ω and circumscribed about a circle Γ. Find $\frac{\text{Area}(\omega)}{\text{Area}(\Gamma)}$.

[A] $\frac{3}{10}$ [B] $\frac{4}{15}$ [C] $\frac{1}{4}$ [D] $\frac{5}{18}$

Q-11 [q11]⟩ A fair die is rolled 20 times. What is the probability that the sum of the numbers obtained is strictly larger than 25 ?

[A] $1 - \frac{\binom{25}{5}}{6^{20}}$ [B] $1 - \frac{\binom{24}{5}}{6^{20}}$ [C] $\frac{\binom{24}{5}}{6^{20}}$ [D] $\frac{\binom{25}{5}}{6^{20}}$

Q-12 [q12]⟩ A given equilateral triangle of side 10 is divided into 100 equilateral triangles of side 1 by drawing parallel lines to the sides of the original triangle. Find the number of equilateral triangles, having vertices in the intersection points of parallel lines whose sides lie on the parallel lines.

A 305 B 310 **C** 315 D 320

Q-13 [q13]⟩ Let A be the set of sequences $\{a_1, a_2, \ldots, a_{2025}\}$ where each $a_i \in \{-1, 1\}$ and satisfies the condition

$$\sum_{i=0}^{44} a_{i^2+1} a_{i^2+2} \ldots a_{(i+1)^2} = 1.$$

Find $\nu_2(|A|)$, where $\nu_2(m)$ denotes the highest power of 2 dividing m.

A 1947 B 1967 **C** 1983 D 2011

Q-14 [q14]⟩ Suraj buys N chocolates for his friend Kavya's birthday party. Kavya decides to divide the chocolates arbitrarily into 10 boxes such **that no box remains empty**. After distributing the chocolates, Kavya notices something fascinating: No matter how she arranges the chocolates in the 10 boxes, there always exist two disjoint sets of boxes such that the total number of chocolates in those sets is the same. Intrigued, Suraj points out: "This fascinating property wouldn't hold if I had brought $N + 1$ chocolates instead." What is the sum of digits of N in the decimal representation?

A 5 B 6 C 7 D 8

Q-15 [q15]⟩ Let ω be the circumcircle of $\triangle ABC$ and O be its circumcenter. Let N be the midpoint of arc BAC in ω, and let I be the incenter of $\triangle ABC$. Suppose J is a point such that $\triangle ABJ \sim \triangle AJC$. Let Γ be a circle internally tangent to ω at D and to AB and AC at F and E, respectively. Given that $\angle NIO = 90°$, $AJ = 10$, and the perimeter of $\triangle ABC$ is 25, find $\dfrac{[DEF]}{[ABC]}$.

A $\frac{2}{5}$ B $\frac{4}{5}$ C $\frac{8}{25}$ **D** $\frac{16}{25}$

Section 2: Objective Questions

This section contains **5 objective questions**, each with **exactly one correct answer**. Each correct response will be awarded $+\mathbf{5}$ marks, and each incorrect response will incur a penalty of $-\mathbf{1.25}$ marks. Candidates must darken **exactly one square in the OMR sheet** per question using a black or blue ball pen.

Q-16 [q1]⟩ Alice and Bob are playing a game called **Avoiding Fifteen**. The game follows these rules:

1. The players take turns selecting a number from 1 to 9 and adding it to a shared pool.
2. Each number can be chosen only once.
3. The game continues until a player is forced to form a set of three numbers in the pool that sum to 15. If this happens, that player loses the game immediately.

For Example- Suppose the shared pool contains 4, 9, *and* 3. *Now, it's Alice's turn, and she selects* 5. *At this point, Bob must choose from* $1, 2, 6, 7$ *or* 8 *since other numbers are already taken. However, no matter which number Bob picks and puts it inside the lot, at least one of the following* $9+5+1 = 4+9+2 = 4+5+6 = 3+5+7 = 4+3+8 = 15$ *will gives three numbers in lot which add up to* 15. *Since Bob is forced to create a set that sums to* 15*, he immediately loses the game.*
Assuming Alice plays first, consider the following statements

1. Statement I: Alice has a winning strategy.
2. Statement II: Neither Alice Nor Bob has a winning strategy.
3. Statement III: If Alice selects 4, then Bob has a winning strategy.

(A) Only I is correct.
(B) Only II is correct.
(C) I and III are correct, II is incorrect.
(D) II and III are correct, I is incorrect.

Q-17 [q2]⟩ Alice and Bob are playing a game called **Finding Fifteen**, where they take turns selecting numbers from 1 to 9, ensuring that no number is chosen more than once. Each player maintains their own separate collection of chosen numbers. The objective is to be the first to collect three numbers that add up to 15. If all nine numbers are picked and neither player forms such a sum, the game ends in a draw.
Consider the following statements

1. Statement I: Alice has a winning strategy.
2. Statement II: Neither Alice Nor Bob has a winning strategy.
3. Statement III: If Alice selects 4 and the Bob selects 3, then Alice has a winning strategy.

Which of the following options are correct.

(A) Only I is correct.
(B) Only I is correct.
(C) I and III are correct, II is incorrect.
(D) II and III are correct, I is incorrect.

Q-18 [q3]⟩ In a **scalene** $\triangle ABC$, let ω be the circumcircle with $AB = 72$, $BC = 85$. Define I as its incenter and E, F as midpoints of CA, AB. Let M be the midpoint of arc BC of ω (not containing A). Given that the circle Γ centered at I passes through E, F, find the length of the tangent from L to Γ.

A 42 B 51 C 54 D 68

Q-19 [q4]⟩ Let p be an odd prime number, and let a_i for $i = 1, 2, \ldots, p-1$ be the Legendre symbol of i relative to p, i.e.,

$$a_i = \begin{cases} 1, & \text{if } i^{\frac{p-1}{2}} \equiv 1 \pmod{p}, \\ -1, & \text{otherwise.} \end{cases}$$

Consider the polynomial

$$f(X) = a_1 + a_2 X + \cdots + a_{p-1} X^{p-2}.$$

We have the following statements:

(i) 1 is a simple root of $f(X)$ if and only if $p \equiv 3 \pmod{5}$.

(ii) If $p \equiv 5 \pmod{8}$, then 1 is a root of order exactly two for $f(X)$.

Which statements are true?

A (i) and (ii) B Only (i) C None are true D Only (ii)

Q-20 [q5]⟩ Assume a, b, c and d are real numbers satisfying

$$a^2 + b^2 + c^2 + d^2 = a + b + c + d$$

Then the maximum possible value of

$$\left| \left(a - \frac{1}{2}\right)\left(b - \frac{1}{2}\right)^2 \left(c - \frac{1}{2}\right)\left(d - \frac{1}{2}\right)^2 \right|$$

is

A $\frac{1}{32}$ B $\frac{1}{27}$ C $\frac{1}{64}$ D $\frac{1}{54}$

Section 3: Numerical Answer Type Questions

This section comprises **6 numerical answer type questions**, where the answer is an **integer between** 000 **and** 999 **(both inclusive)**. Each correct response will be awarded **+5** marks, and no negative marking will be applied. Candidates must darken **three squares, one for each digit, in the OMR sheet**, for every question using a black or blue ball pen. For example, the number 9 should be marked as 009, 10 as 010, and 831 as 831.

Q-21 [nq1]⟩ Let $\triangle ABC$ be a right triangle with $\angle ABC = 90°$, $AB = 5$, and $BC = 12$. Let D and E be points on segments BC and CA, respectively, such that $\frac{BD}{DC} = \frac{1}{5}$ and $\frac{AE}{EC} = \frac{1}{4}$. Let P be a point on segment BE such that $\angle APE = 45°$. If the the angle between the tangent at D to circle passing through A, P and D and BC is α, compute $780\sin^2(\alpha)$.

Q-22 [nq2]⟩ Find the smallest integer n for which the set $A = \{n, n+1,\ n+2, \ldots, 2n\}$ contains five elements $a < b < c < d < e$ so that

$$\frac{a}{c} = \frac{b}{d} = \frac{c}{e}.$$

is ??

Q-23 [nq3]⟩ Let E and F be the midpoints of sides CA and AB of $\triangle ABC$, and let O be its circumcenter. Let its incircle touch BC at D. Define h_b and h_c to be the lengths of the altitudes from B and C onto CA and AB, respectively. Suppose $AO = 20 = EO + FO$ and $\frac{1}{h_b} + \frac{1}{h_c} = \frac{5}{64}$. Calculate OD.

Q-24 [nq4]⟩ Let α be the maximum value of $x^2+y^2+z^2$ under the condition $x^2+2x+y^2+4x+z^2+8z = -20$. Assume x, y and z are reals. Suppose α is of the form $a + b\sqrt{c}$, where a, b, c are integers. Return $(\alpha - a)^2$

Q-25 [nq5]⟩ Consider an infinite chessboard with an unconventional knight such that it moves m moves in one direction and n moves in the perpendicular direction. Call such a knight (m, n) knight if $m \leq n$. A knight is *spanning* if it can reach every square on the board in finite number of moves. Find number of m's such that a $(m, 2025)$ knight is *spanning*.

Q-26 [nq6]⟩ Let $n = 9$. Consider a sequence of integers $a_1, a_2, a_3, \ldots, a_n$ satisfying the condition

$$\sum_{i=1}^{k} (-1)^i a_i \equiv 0 \mod (k) \text{ for all } k = 1, 2, 3 \ldots n$$

Define the sequence $\{b_k\}$ by

$$b_k = n! \sum_{i=k}^{n} \frac{(-1)^{i-k}}{i}$$

Let the set $S \subset \mathbb{Z}$ be the set of all possible sums of the form $\sum_{i=1}^{n} b_i a_i$. Let g denote the smallest natural number in S. Determine the last three digits of $\frac{g}{5}$.

End of Question Paper

Space for Rough Work

Space for Rough Work

Optical Mark Recognition (OMR) Sheet

IIT Bombay Mathematics Olympiad 2025(IITBMO2025)

A

Name: .....

Reg. No.: M O 2 5 [ ] [ ] [ ] A Class: [ ]

Stud. Sign.: .....

Invig. Sign.: .....

Class

9 10

- Bubble your class and registration number in the designated area.
- For numerical answer, carefully fill in the correct digit in each column. If your answer is 073, you must darken the first digit 0 (that is, Hundreds digit) in the first column, the second digit 7 (that is, Tens digit) in the second column, and the third digit 3 (that is, Ones digit) in the third column.

Reg. No.

0	0	0
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	8	8
9	9	9

Mark Your Numerical Answer for **Section 3** Questions

Q-21⟩	Q-22⟩	Q-23⟩	Q-24⟩	Q-25⟩	Q-26⟩
3	1, 6	1, 2	2, 8	4, 5	5, 7, 6

Mark Your Choice for **Section 1** and **Section 2** Questions

Q-01⟩		B			Q-08⟩			C		Q-15⟩				D	
Q-02⟩		B			Q-09⟩				D	Q-16⟩	A				
Q-03⟩	A				Q-10⟩		B			Q-17⟩				D	
Q-04⟩		B			Q-11⟩	A				Q-18⟩	A				
Q-05⟩		B			Q-12⟩			C		Q-19⟩				D	
Q-06⟩				D	Q-13⟩			C		Q-20⟩				D	
Q-07⟩		B			Q-14⟩	A									