Mathematics Olympiad 2025

Mathematics Association IIT Bombay

Name: .....

Reg. No.: M O 2 5 _ _ _ B Level: II

Code: A

Date: Feb 9, 2025

Maximum Marks: 100

Time: 09:30 - 11:30

Read the following instructions carefully before attempting the question paper.

- Do not open this booklet until instructed by the invigilators.
- Questions must be answered on **O**ptical **M**ark **R**ecognition sheet (**OMR**) by darkening the appropriate squares (marked A, B, C, D or $0-9$) using blue/black ball pen.
- Write your registration number, name, and the Question Paper code (**A**, **B**, **C** or **D**) in the space provided on the OMR sheet.
- The question paper consists of three sections: **Section 1**, **Section 2** and **Section 3**.
- **Section 1:** This section consists of **15 objective questions**, each having **exactly one correct answer**. Each correct response will be awarded $+\mathbf{3}$ marks, while each incorrect response will result in a penalty of $-\mathbf{0.75}$ marks. Candidates must darken **exactly one square in the OMR sheet** per question using a black or blue ball pen.
- **Section 2:** This section contains **5 objective questions**, each with **exactly one correct answer**. Each correct response will be awarded $+\mathbf{5}$ marks, and each incorrect response will incur a penalty of $-\mathbf{1.25}$ marks. Candidates must darken **exactly one square in the OMR sheet** per question using a black or blue ball pen.
- **Section 3:** This section comprises **6 numerical answer type questions**, where the answer is an **integer between** 000 **and** 999 **(both inclusive)**. Each correct response will be awarded $+\mathbf{5}$ marks, and no negative marking will be applied. Candidates must darken **three squares, one for each digit, in the OMR sheet**, for every question using a black or blue ball pen. For example, the number 9 should be marked as 009, 10 as 010, and 831 as 831.
- You won't be allowed to leave the examination hall during the exam.

Notation

- For a set A, the notation $x \in A$ means "x is an element of the set A". The notation $|A|$ means the cardinality of set A.
- $A \setminus B$ means the set difference: elements in A but not in B.
- $\mathbb{N}$ = the set of all natural numbers = $\{1, 2, 3, \ldots\}$.
- $\mathbb{Z}$ = the set of all integers = $\{0, -1, 1, -2, 2, \ldots\}$.
- $\mathbb{R}$ = the set of all real numbers.
- For $a, b \in \mathbb{R}$,
  - (i) $[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$.
  - (ii) $[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$.
  - (iii) $(a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$.
  - (iv) $(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$.
  - (v) $a \mid b$ means "a divides b".
  - (vi) $a \equiv b \pmod{k}$ means "k divides $(a - b)$".
- For a natural number n,
$$n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1.$$
- For a sequence $a_1, a_2, \ldots$ and two natural numbers $m < n$,
$$\prod_{i=m}^{n} a_i = a_m \cdot a_{m+1} \cdots a_{n-1} \cdot a_n.$$
- For a real number x, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.
- $[ABC]$ = Area of triangle ABC.
- $\odot(PQR)$ = Circle passing through P, Q, R.
- $f^{(k)}(x)$ means "kth derivative of function f".

Table of Values

Constant	Value
e	2.718
π	3.142
$\ln 2$	0.693
$\ln 3$	1.099
$\ln 5$	1.609
$\ln 7$	1.946

Section 1: Objective Questions

This section consists of **15 objective questions**, each having **exactly one correct answer**. Each correct response will be awarded **+3** marks, while each incorrect response will result in a penalty of **−0.75** marks. Candidates must darken **exactly one square in the OMR sheet** per question using a black or blue ball pen.

Q-1 [q01]⟩ Let the sum of all natural numbers n for which $\sum_{k=1}^{S(n)} k^3 = n$ be T. Find the sum of the distinct prime divisors of T. Here $S(k)$ denotes the sum of digits of k in its decimal representation.

A 537 B 1015 C 1815 D 2037

Q-2 [q02]⟩ Let $n = 8$. Consider a sequence of integers $a_1, a_2, a_3, \ldots, a_n$ satisfying the condition

$$\sum_{i=1}^{k} (-1)^i a_i \equiv 0 \mod (k) \text{ for all } k = 1, 2, 3 \ldots n$$

Define the sequence $\{b_k\}$ by

$$b_k = n! \sum_{i=k}^{n} \frac{(-1)^{i-k}}{i}$$

Let the set $S \subset \mathbb{Z}$ be the set of all possible sums of the form $\sum_{i=1}^{n} b_i a_i$. Let g denotes the smallest natural number in S, determine the last two digits of $\frac{g}{5}$.

A 28 B 64 C 32 D 16

Q-3 [q03]⟩ Assume a_1, a_2, a_3, a_4 and a_5 are complex numbers satisfying

$$\sum_{i=1}^{5} {a_i}^k = 5(k+1)$$

Define S_i as the sum of all distinct products of i different a_j's (with each k_i representing a distinct index):

$$S_i = \sum_{1 \le k_1 < k_2 < \cdots < k_i \le 5} a_{k_1} a_{k_2} \ldots a_{k_i}.$$

then determine

$$4S_2 - 3S_3 + 2S_4 - S_5$$

A 22 B -22 C 44 D -44

Q-4 [q04]⟩ F is a polynomial, expressed like follows

$$F(x) = p_n x^n + p_{n-1} x^{n-1} \dots p_1 x + p_0$$

where $n > 2$ and the p_i are primes not all equal. Which of the following is always false

A F has a negative real root

B F has the root of the form $\alpha^4 + 64$ for some integer α

C F is irreducible over integers

D F has a root of the form $n^2 + 1$ where n is a positive integer.

Q-5 [q05]⟩ The numbers $1, 2, \dots, 16$ are written on a blackboard. You perform the following operation five times:

1. Group all the remaining numbers into pairs.
2. For each pair, replace the pair with the absolute value of the difference of the two numbers.

At the end of four iterations, only one number remains on the blackboard. What is the number of possible values at the end of the process?

A 6 B 8 C 12 D 16

Q-6 [q06]⟩ Let n be a positive integer and a, b, c be real numbers such that $a^n = a + b$, $b^n = b + c$ and $c^n = c + a$. Then which of the following statements is true?

A $a = b = c$ B $a - c = 2b$ C $a^2 + b^2 = c^2$ D $a^3 + b^3 = c^3$

Q-7 [q07]⟩ For $x_n, n \geqslant 2$, where $x_1 = 1$ and

$$4\left(x_1 x_n + 2x_2 x_{n-1} + 3x_3 x_{n-2} + \cdots + n x_n x_1\right) = (n+1)\left(x_1 x_2 + x_2 x_3 + \cdots + x_n x_{n+1}\right)$$

, for each $n \geqslant 1$. You may want to refer to the table of values. Then

$$\sum_{i=1}^{2025^2} \frac{1}{x_i}$$

is

A 16.88 B 15.22 C 14.88 D 13.44

Q-8 [q08]⟩ Let $ABCD$ be a kite with diagonals of lengths 2 and 3. Moreover, suppose $ABCD$ is both inscribed in a circle ω and circumscribed about a circle Γ. Find $\dfrac{\text{Area}(\omega)}{\text{Area}(\Gamma)}$.

A $\frac{3}{10}$ B $\frac{4}{15}$ C $\frac{1}{4}$ D $\frac{5}{18}$

Q-9 [q09]⟩ Let α be the maximum value of $x^2 + y^2 + z^2$ under the condition $x^2 + 2x + y^2 + 4x + z^2 + 8z = -20$. Assume x, y and z are reals. Suppose α is of the form $a + b\sqrt{c}$, where a, b, c are integers. Return $(\alpha - a)^2$

A 882 B 722 C 648 D 578

Q-10 [q10]⟩ Let

$$A(n) = \{f : [n] \to [n] \mid f(f(x)) + f(x) = 5 \text{ for all } x \in [n]\}.$$

Find the sum of all primes p such that p divides $|A(p)|$. (Here, $[n]$ is the set $\{1, 2, \ldots, n\}$.)

A 16 **B** 18 C 22 D 29

Q-11 [q11]⟩ Consider all 3-element subsets of the set $\{1, 2, \cdots, 2025\}$. Choose the second largest element from each subset. The arithmetic mean of all these second largest elements is $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

A 2027 B 2025 C 1015 **D** 1014

Q-12 [q12]⟩ Suppose A, B, C, D, E, and F are six friends who attended a dinner party and sat around a round table. Upon returning, they could hardly remember their exact positions but recalled the following: A was not sitting with B and C, B was not sitting with E and F, C was uncertain about their position, D was not sitting with E, E was not sitting with A, and F was uncertain about their position. Determine which option is correct:

A D and E are sitting together
B C and D are sitting together
C C and B are sitting together
D D and F are sitting together

Q-13 [q13]⟩ Call a sequence $\{s_n\}$ of positive integers *sagacious* if it satisfies $s_1 = 2$, $s_2 = 3$, and for $k \geq 1$,

$$(s_{k+2} - 2s_k)(s_{k+2} - 3s_{k+1} + 2s_k). = 0$$

For how many positive integers $M \leq 2025$, does there exist a *sagacious* sequence $\{s_n\}$ and an $m \in \mathbb{N}$ such that $s_m = M$?

A 85 **B** 65 C 55 D 45

Q-14 [q14]⟩ Let $P(x) = x^3 + mx^2 + (2n + 1)x + n$ be a polynomial with positive real roots. Consider the following statements:

- **Statement A:** $m \leq -\frac{5}{4}$
- **Statement B:** $m \geq -18$
- **Statement C:** $m \geq 18n$
- **Statement D:** $m \leq 10n$

How many of above statements always hold true?

A 0 B 1 **C** 2 D 3

Q-15 [q15]⟩ Assume that

$$\sum_{i=1}^{n} i^k = \sum_{i=1}^{k+1} a_{i,k} n^i$$

Then the value of

$$\sum_{j=6}^{9} \binom{10}{j} a_{7,j}$$

A 120 B 140 C 160 D 180

Section 2: Objective Questions

This section contains **5 objective questions**, each with **exactly one correct answer**. Each correct response will be awarded **+5** marks, and each incorrect response will incur a penalty of **−1.25** marks. Candidates must darken **exactly one square in the OMR sheet** per question using a black or blue ball pen.

Q-16 [q1]⟩ Alice and Bob are playing a game called **Avoiding Fifteen**. The game follows these rules:

1. The players take turns selecting a number from 1 to 9 and adding it to a shared pool.
2. Each number can be chosen only once.
3. The game continues until a player is forced to form a set of three numbers in the pool that sum to 15. If this happens, that player loses the game immediately.

For Example- Suppose the shared pool contains 4, 9, *and* 3. *Now, it's Alice's turn, and she selects* 5. *At this point, Bob must choose from* $1, 2, 6, 7$ *or* 8 *since other numbers are already taken. However, no matter which number Bob picks and puts it inside the lot, at least one of the following* $9+5+1 = 4+9+2 = 4+5+6 = 3+5+7 = 4+3+8 = 15$ *will gives three numbers in lot which add up to* 15. *Since Bob is forced to create a set that sums to* 15, *he immediately loses the game.*
Assuming Alice plays first, consider the following statements

1. Statement I: Alice has a winning strategy.
2. Statement II: Neither Alice Nor Bob has a winning strategy.
3. Statement III: If Alice selects 4, then Bob has a winning strategy.

(A) Only I is correct.
(B) Only II is correct.
(C) I and III are correct, II is incorrect.
(D) II and III are correct, I is incorrect.

Q-17 [q2]⟩ Alice and Bob are playing a game called **Finding Fifteen**, where they take turns selecting numbers from 1 to 9, ensuring that no number is chosen more than once. Each player maintains their own separate collection of chosen numbers. The objective is to be the first to collect three numbers that add up to 15. If all nine numbers are picked and neither player forms such a sum, the game ends in a draw.
Consider the following statements

1. Statement I: Alice has a winning strategy.
2. Statement II: Neither Alice Nor Bob has a winning strategy.
3. Statement III: If Alice selects 4 and the Bob selects 3, then Alice has a winning strategy.

Which of the following options are correct.

(A) Only I is correct.
(B) Only I is correct.
(C) I and III are correct, II is incorrect.
(D) II and III are correct, I is incorrect.

Q-18 [q3]⟩ Let ω be the circumcircle of $\triangle ABC$ with circumcenter O. Let N be the midpoint of arc BAC in ω, and let I be the incenter of $\triangle ABC$. Suppose J is a point inside ω such that $\triangle ABJ \sim \triangle AJC$. Let Γ be a circle internally tangent to ω at D and to AB and AC at F and E, respectively. Given that $\angle NIO = 90°$, $AJ = 10$, and the perimeter of $\triangle ABC$ is 25, find $\dfrac{[DEF]}{[ABC]}$.

A $\frac{8}{25}$ B $\frac{16}{25}$ C $\frac{2}{5}$ D $\frac{4}{5}$

Q-19 [q4]⟩ In a **scalene** $\triangle ABC$, let ω be the circumcircle with $AB = 72$, $BC = 85$. Define I as its incenter and E, F as midpoints of CA, AB. Let M be the midpoint of arc BC of ω (not containing A). Given that the circle Γ centered at I passes through E, F, find the length of the tangent from L to Γ.

A 42 B 51 C 54 D 68

Q-20 [q5]⟩ A number is defined as a Deck number if its digits form a permutation of the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Examples of Deck numbers include:

- 1027348659
- 0217348659
- 123456789, which is equivalent to 0123456789 (as leading zeros are permitted in this definition).

Determine the number of Deck numbers such that their double (i.e., 2× the number) is also a Deck number.

A $2^5 5!5!$ B $2^4 5!5!$ C $2^5 5!4!$ D $2^4 5!4!$

Section 3: Numerical Answer Type Questions

This section comprises **6 numerical answer type questions**, where the answer is an **integer between** 000 **and** 999 **(both inclusive)**. Each correct response will be awarded +**5** marks, and no negative marking will be applied. Candidates must darken **three squares, one for each digit, in the OMR sheet**, for every question using a black or blue ball pen. For example, the number 9 should be marked as 009, 10 as 010, and 831 as 831.

Q-21 [nq1]⟩ Find smallest $n \in \mathbb{N}$ so that:

$$\left| \int_{\pi}^{3\pi/2} \frac{\sin x}{x} dx \right| < \frac{n}{10}$$

Q-22 [nq2]⟩ Let $a, b \in \mathbb{C}$ such that $|a| + |b| = 1$. Let the sum of the maximum and minimum possible values that the expression $|a + 2b| + |2a + 3b|$ attains be S. If $S = \frac{p}{q}$, where p, q are relatively prime positive integers, find $p + q$.

Q-23 [nq3]⟩ Let E and F be the midpoints of sides CA and AB of $\triangle ABC$, and let O be its circumcenter. Let its incircle touch BC at D. Define h_b and h_c to be the lengths of the altitudes from B and C onto CA and AB, respectively. Suppose $AO = 20 = EO + FO$ and $\frac{1}{h_b} + \frac{1}{h_c} = \frac{5}{64}$. Calculate the length of OD.

Q-24 [nq4]⟩ Let

$$r_k = \frac{\int_0^1 \left(1 - x^{2025}\right)^k \, dx}{\int_0^1 \left(1 - x^{2025}\right)^{k+1} \, dx}, \quad k \in \mathbb{N}.$$

Let

$$G = \sum_{k=1}^{63} \frac{1}{2025\,(r_k - 1)} - \sum_{k=1}^{10} k + 1$$

Then the last three digits of G are equal to?

Q-25 [nq5]⟩ Assume a, b, c and d are real numbers satisfying

$$a^2 + b^2 + c^2 + d^2 = a + b + c + d$$

Then the minimum possible value of

$$\left| \left(a - \frac{1}{2}\right)\left(b - \frac{1}{2}\right)^2 \left(c - \frac{1}{2}\right)\left(d - \frac{1}{2}\right)^2 \right|^{-1}$$

is

Q-26 [nq6]⟩ Seven people line up to board an airplane, each with a boarding pass indicating an assigned seat. However, the first person has lost their boarding pass and chooses a seat at random. Every subsequent person sits in their assigned seat if it is available; otherwise, they choose an unoccupied seat at random. Given that the probability that the last person sits in their assigned seat is $\frac{6}{M}$. Then determine the value of M.

End of Question Paper

Space for Rough Work

Optical Mark Recognition (OMR) Sheet

IIT Bombay Mathematics Olympiad 2025(IITBMO2025)

A

Name: .....

Reg. No.: M O 2 5 [ ] [ ] [ ] B Class: [ ]

Stud. Sign.: .....

Invig. Sign.: .....

Class

11 12

- Bubble your class and registration number in the designated area.
- For numerical answer, carefully fill in the correct digit in each column. If your answer is 073, you must darken the first digit 0 (that is, Hundreds digit) in the first column, the second digit 7 (that is, Tens digit) in the second column, and the third digit 3 (that is, Ones digit) in the third column.

Reg. No.

0	0	0
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	8	8
9	9	9

Mark Your Numerical Answer for **Section 3** Questions

Q-21⟩	Q-22⟩	Q-23⟩	Q-24⟩	Q-25⟩	Q-26⟩
3	3	2	5	4	2

Mark Your Choice for **Section 1** and **Section 2** Questions

Q-01⟩	A	**B**	C	D	Q-08⟩	A	**B**	C	D	Q-15⟩	**A**	B	C	D
Q-02⟩	A	**B**	C	D	Q-09⟩	**A**	B	C	D	Q-16⟩	**A**	B	C	D
Q-03⟩	A	B	**C**	D	Q-10⟩	A	**B**	C	D	Q-17⟩	A	B	C	**D**
Q-04⟩	A	**B**	C	D	Q-11⟩	A	B	C	**D**	Q-18⟩	A	**B**	C	D
Q-05⟩	A	**B**	C	D	Q-12⟩	A	B	**C**	D	Q-19⟩	**A**	B	C	D
Q-06⟩	**A**	B	C	D	Q-13⟩	A	**B**	C	D	Q-20⟩	A	**B**	C	D
Q-07⟩	A	**B**	C	D	Q-14⟩	A	B	**C**	D					