MA 108 - Extra notes - NOT FOR YOUR EXAM

Let $D \subseteq \mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$

D is convex : if given $(x_0, y_0), (x_1, y_1) \in D$, the straight line segment joining them lies in D.

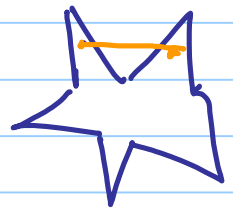
D is star-shaped: if there is a point $(x_0, y_0) \in D$ such that for any $(x, y) \in D$, the straight line segment joining them lies in D.

D is simply-connected : if any simple closed curve in D can be "shrunk" to a point in D.

D convex $\Rightarrow$ D star-shaped $\Rightarrow$ D simply-connected

Note : There implications are strict as can be seen from the examples below.

1) Star-shaped $\not\Rightarrow$ convex



2) Simply-connected $\not\Rightarrow$ star-shaped



Fact: If D is simply-connected then closed 2 forms are exact.