MA 108——Tutorial Sheet No. 1

Q.1. Classify the following equations (order, linear or non-linear):
(i) $\frac{d^3y}{dx^3} + 4(\frac{dy}{dx})^2 = y$ (ii) $\frac{dy}{dx} + 2y = \sin x$ (iii) $y\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 0$
(iv) $\frac{d^4y}{dx^4} + (\sin x)\frac{dy}{dx} + x^2y = 0$. (v)$(1+y^2)\frac{d^2y}{dt^2} + t\frac{d^6y}{dt^6} + y = e^t$.

Q.2. Formulate the differential equations represented by the following functions by eliminating the arbitrary constants a, b and c:
(i) $y = ax^2$ (ii) $y - a^2 = a(x-b)^2$ (iii) $x^2 + y^2 = a^2$ (iv) $(x-a)^2 + (y-b)^2 = a^2$
(v) $y = a\sin x + b\cos x + a$ (vi) $y = a(1-x^2) + bx + cx^3$ (vii) $y = cx + f(c)$.
Also state the order of the equations obtained.

Q.3. Solve the equation $x^3(\sin y)y' = 2$. Find the particular solution such that $y(x) \to \frac{\pi}{2}$ as $x \to +\infty$.

Q.4. Prove that a curve with the property that all its normals pass through a point is a circle.

Q.5. Let φ_i be a solution of $y' + ay = b_i(x)$ for $i = 1, 2$.
Show that $\varphi_1 + \varphi_2$ satisfies $y' + ay = b_1(x) + b_2(x)$. Use this result to find the solutions of $y' + y = \sin x + 3\cos 2x$ passing through the origin.

Q.6. Obtain the solution of the following differential equations:
(i) $(x^2+1)dy + (y^2+4)dx = 0$; $y(1) = 0$ (ii) $y' = y\cot x$; $y(\pi/2) = 1$
(iii) $y' = y(y^2-1)$, with $y(0) = 2$ or $y(0) = 1$, or $y(0) = 0$ (iv) $(x+2)y' - xy = 0$; $y(0) = 1$
(v) $y' + \dfrac{y-x}{y+x} = 0$; $y(1) = 1$ (vi) $y' = (y-x)^2$; $y(0) = 2$
(vii) $2(y\sin 2x + \cos 2x)dx = \cos 2x dy$; $y(\pi) = 0$. (viii) $y' = \dfrac{1}{(x+1)(x^2+1)}$

Q.7. For each of the following differential equations, find the general solution (by substituting $y = vx$)
(i) $y' = \dfrac{y^2 - xy}{x^2 + xy}$ (ii) $x^2y' = y^2 + xy + x^2$
(iii) $xy^{'} = y + x\cos^2(y/x)$ (iv) $xy' = y(\ln y - \ln x)$

Q.8. Show that the differential equation $\dfrac{dy}{dx} = \dfrac{ax+by+m}{cx+dy+n}$ where a, b, c, d, m and n are constants can be reduced to $\dfrac{dy}{dt} = \dfrac{ax+by}{cx+dy}$ if $ad - bc \neq 0$. Then find the general solution of
(i) $(1 + x - 2y) + y'(4x - 3y - 6) = 0$
(ii) $y' = \frac{y-x+1}{y-x+5}$
(iii) $(x + 2y + 3) + (2x + 4y - 1)y' = 0$.

Q.9. Solve the differential equation $\sqrt{1-y^2}dx + \sqrt{1-x^2}dy = 0$ with the conditions $y(0) = \dfrac{\pm 1}{2}\sqrt{3}$. Sketch the graphs of the solutions and show that they are each arcs of the same ellipse. Also show that after these arcs are removed, the remaining part of the ellipse does not satisfy the differential equation.

Q.10. State the conditions under which the following equations are exact.

(i) $[f(x) + g(y)]dx + [h(x) + k(y)]dy = 0$

(ii) $(x^3 + xy^2)dx + (ax^2y + bxy^2)dy = 0$

(iii) $(ax^2 + 2bxy + cy^2)dx + (bx^2 + 2cxy + gy^2)dy = 0$

Q.11. Solve the following exact equations

(i) $3x(xy - 2)dx + (x^3 + 2y)dy = 0$ (ii) $(\cos x \cos y - \cot x)dx - \sin x \sin y dy = 0.$

(iii) $e^x y(x + y)dx + e^x(x + 2y - 1)dy = 0$