Tutorial Sheet No. 4

Q.1. Solve the following initial value problems.

(i) $(D^2+5D+6)y=0, \quad y(0)=2, y'(0)=-3$ (ii) $(D+1)^2y=0, \quad y(0)=1, y'(0)=2$

(iii) $(D^2+2D+2)y=0, \quad y(0)=1, y'(0)=-1$

Q.2. Solve the following initial value problems.

(i) $(x^2D^2-4xD+4)y=0, y(1)=4, y'(1)=1$

(ii) $(4x^2D^2+4xD-1)y=0, y(4)=2, y'(4)=-1/4$

(iii) $(x^2D^2-5xD+8)y=0, y(1)=5, y'(1)=18$

Q.3. Using the Method of Undetermined Coefficients, determine a particular solution of the following equations. Also find the general solutions of these equations.

(i) $y''+2y'+3y=27x$ (ii) $y''+y'-2y=3e^x$

(iii) $y''+4y'+4y=18\cos x$

(iv) $y''+4y'+3y=\sin x+2\cos x$ (v) $y''-2y'+2y=2e^x\cos x$

(vi) $y''+y=x\cos x+\sin x$

Q.4. Solve the following initial value problems.

(i) $y''+y'-2y=14+2x-2x^2, y(0), y'(0)=0.$

(ii) $y''+y'-2y=-6\sin 2x-18\cos 2x;\ y(0)=2, y'(0)=2.$

(iii) $y''-4y'+3y=4e^{3x}, y(0)=-1, y'(0)=3.$

Q.5. For each of the following equations, write down the form of the particular solution. Do not go further and compute the Undetermined Coefficients.

(i) $y''+y=x^3\sin x$ (ii) $y''+2y'+y=2x^2e^{-x}+x^3e^{2x}$ (iii) $y'+4y=x^3e^{-4x}$.

Q.6. Using the Method of Variation of Parameters, determine a particular solution for each of the following.

(i) $y''-5y'+6y=2e^x$ (ii) $y''+y=\tan x, 0<x<\dfrac{\pi}{2}$

(iii) $y''+4y'+4y=x^{-2}e^{-2x}, x>0$ (iv) $y''+4y=3\ \text{cosec}\ 2x, 0<x<\dfrac{\pi}{2}$

(v) $x^2y''-2xy'+2y=5x^3\cos x$ (vi) $xy''-y'=(3+x)x^3e^x$

Q.7. Prove that the the functions $e^{r_1x}, e^{r_2x}, \ldots, e^{r_nx}$, where $r_1, r_2, \ldots, r_n$ are distinct real numbers, are linearly independent.

Q.8. For the following nonhomogeneous equations, a solution y_1 of the corresponding homogeneous equation is given. Find a second solution y_2 of the corresponding homogeneous equation and the general solution of the nonhomogeneous equation using the Method of Variation of Parameters.

(i) $(1+x^2)y''-2xy'+2y=x^3+x, \quad y_1=x$ (ii) $xy''-y'+(1-x)y=x^2, \quad y_1=e^x$

(iii) $(2x+1)y''-4(x+1)y'+4y=e^{2x}, \quad y_1=e^{2x}$

(iv) $(x^3-x^2)y''-(x^3+2x^2-2x)y'+(2x^2+2x-2)y=(x^3-2x^2+x)e^x, \quad y_1=x^2$

Q.9. A linear DE with constant coefficients has characteristic polynomial $f(x) = 0$. If all the roots of $f(x)$ are negative, prove that every solution of the differential equation approaches 0 as $x \to +\infty$. What can you conclude about the behaviour of all solutions in the interval $[0, +\infty)$, if all the roots of $f(x)$ are non-positive?

Q.10. In each case, find a linear differential equation with constant coefficients satisfied by all the given functions.

(i) $u_1(x) = e^x$, $u_2(x) = e^{-x}$, $u_3(x) = e^{2x}$, $u_4(x) = e^{-2x}$.

(ii) $u_1(x) = e^{-2x}$, $u_2(x) = xe^{-2x}$, $u_3(x) = x^2e^{-2x}$.

(iii) $u_1(x) = 1$, $u_2(x) = x$, $u_3(x) = e^x$, $u_4(x) = xe^x$.
