Tutorial Sheet No. 6

Q.1. Find the Laplace Transform of the following functions.

(i) $t\cos wt$ (ii) $t\sin wt$ (iii) $e^{-t}\sin^2 t$ (iv) t^2e^{-at} (v) $(1+te^{-t})^3$ (vi) $(5e^{2t}-3)^2$

(vii) $te^{-2t}\sin wt$ (viii) t^ne^{at} (ix) $t^2e^{-at}\sin bt$ (xi) $\cosh at\cos at$

Q.2. Find the inverse Laplace transforms of the following functions.

(i) $\dfrac{s^2-w^2}{(s^2+w^2)^2}$ (ii) $\dfrac{2as}{(s^2-a^2)^2}$ (iii) $\dfrac{1}{(s^2+w^2)^2}$ (iv) $\dfrac{s^3}{(s^4+4a^4)}$ (v) $\dfrac{s-2}{s^2(s+4)^2}$ (vi) $\dfrac{1}{s^4-2s^3}$ (vii) $\dfrac{1}{s^4(s^2+\pi^2)}$

(viii) $\dfrac{s^2+a^2}{(s^2-a^2)^2}$ (ix) $\dfrac{s^3+3s^2-s-3}{(s^2+2s+5)^2}$ (x) $\dfrac{s^3-7s^2+14s-9}{(s-1)^2(s-2)^2}$

Q.3. Solve the following intial value problems using Laplace transforms.

(i) $y''+y=\sin 3t;\ y(0)=y'(0)=0$ (ii) $y''+3y'+2y=e^{-t};\ y(0)=y'(0)=0$

(iii) $y''+2y'-8y=0;\ y(0)=1; y'(0)=8$ (iv) $y''+2y'+y=2\cos t;\ y(0)=3, y'(0)=0$

(v) $y''-2y'+5y=8\sin t-4\cos t;\quad y(0)=1; y'(0)=3$

(vi) $y''-2y'-3y=10\sin h2t;\quad y(0)=0; y'(0)=4$

Q.4. Solve the following systems of differential equations using Laplace transforms.

(i) $x'=x+y,\ y'=4x+y$ (ii) $x'=3x+2y,\ y'=-5x+y$

(iii) $x''-x+y'=y=1,\ y''+y+x'-x=0$ (iv) $x'=5x+8y+1,\ y'=-6x-9y+t,\ x(0)=4,\ y(0)=-3$

(v) $y_1'+y_2=2\cos t;\ y_1+y_2'=0; y_1(0)=0; y_2(0)=1$

(vi) $y_1''+y_2=-5\cos 2t;\ y_2''+y_1=5\cos 2t;\ y_1(0)=1, y_1'(0)=1, y_2(0)=-1, y_2'(0)=1$

(vii) $2y_1'-y_2'-y_3'=0; y_1'+y_2'=4t+2;\ y_2'+y_3=t^2+2, y_1(0)=y_2(0)=y_3(0)=0$

(viii) $y_1''=y_1+3y_2; y_2''=4y_1-4e^t;\ y_1(0)=2; y_1'(0)=3, y_2(0)=1, y_2'(0)=2$

Q.5. Find the Laplace transform of the following periodic functions.

(i) $f(t), f(t+p)=f(t)$ for all $t>0$ and $f(t)$ piecewise continuous

(ii) $f(t)=|\sin wt|$

(iii) $f(t)=1(0<t<\pi); f(t)=-1(\pi<t<2\pi);\ f(t+2\pi)=f(t)$

(iv) $f(t)=t(0\le t\le 1), f(t)=2-t(1\le t\le 2); f(t+2)=f(t)$

(v) $f(t)=\sin t(0\le t\le \pi), f(t)=0(\pi\le t\le 2\pi); f(t+2\pi)=f(t)$

Q.6. Find the Laplace Transform of $f(t)$ where $f(t)=n,\ n-1\le t\le n, n=1,2,3,\ldots$

Q.7. Find $f(t)$ given $\mathcal{L}[f(t)]=(e^{-s}-e^{-2s}-e^{-3s}+e^{-4s})/s^2$

Q.8. Find a real general solution of the following nonhomogeneous linear systems.

(i) $y_1'=y_2+e^{3t},\quad y_2'=y_1-3e^{3t}.$

(ii) $y_1'=3y_1+y_2-3\sin 3t,\quad y_2'=7y_1-3y_2+9\cos 3t-16\sin 3t.$

(iii) $y_1'=y_2+6e^{2t},\quad y_2'=y_1-3e^{2t},\quad y_1(0)=11,\quad y_2(0)=0.$

(iv) $y_1' = 5y_2 + 23, \quad y_2' = -5y_1 + 15t, \quad y_1(0) = 1, \quad y_2(0) = -2.$

(v) $y_1' = y_2 - 5\sin t, \quad y_2' = -4y_1 + 17\cos t, \quad y_1(0) = 5, \quad y_2(0) = 2.$

(vi) $y_1' = 5y_1 + 4y_2 - 5t^2 + 6t + 25, \quad y_2' = y_1 + 2y_2 - t^2 + 2t + 4, \quad y_1(0) = 0, \quad y_2(0) = 0.$