EXERCISES 31/07/2019, MA 419
IIT BOMBAY, AUTUMN 2016

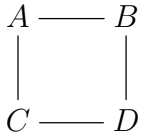
(1) Find a group structure if it exists on a set of three elements $\{x, y, z\}$.

(2) In the following determine whether the systems described are groups. If they are not, point out which of the group axioms fails to hold.
   (a) $G = \mathbb{R}^3$ with dot product.
   (b) $G = \{a_0, a_1, \ldots, a_6\}$, where $a_i.a_j = a_{i+j}$ if $i + j < 7$ and $a_i.a_j = a_{i+j-7}$ if $i + j \geq 7$. (for example, $a_5.a_4 = a_{5+4-7} = a_2$)
   (c) $G = \{A \in M_2(\mathbb{R}) \mid A = \begin{pmatrix} a & b \\ c & c \end{pmatrix}\}$ with matrix additon.
   (d) $G =$ set of all rational numbers with odd denominators when in lowest form , $a.b = a + b$, the usual addition of rational numbers.
   (e) Set of bijections from $\mathbb{N} \to \mathbb{N}$ with respect to composition.
   (f) Set of bijections from $\alpha : \{1, 2, \ldots, n\} \to \{1, 2 \ldots, n\}$ such that $\alpha(1) = 1$.

(3) Heisenberg group over $\mathbb{R}$. Consider the set $H(\mathbb{R}) := \{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \mid a, b, c \in \mathbb{R}\}$. Show that $H(\mathbb{R})$ is a group under the usual matrix multiplication on this set.

(4) Give an example of a group which is not abelian.

(5) Let S denote a square



Consider all possible motions of S which keep the corners and edges in place. Describe this set. Is there a natural binary operation on the set of these rigid motions of a square?