SHARP ENDPOINT L^p–ESTIMATES FOR BILINEAR SPHERICAL MAXIMAL FUNCTIONS

ANKIT BHOJAK

ABSTRACT

The study of maximal averaging operators plays a fundamental role in analysis. For instance, Lebesgue differentiation thereom is a consequence of the $L^p(\mathbb{R}^d)-$boundedness of the classical Hardy-Littlewood maximal function. Another important averaging operator is the spherical maximal function defined as

$$A_* f(x) = \sup_{t>0} \left| \int_{\mathbb{S}^{d-1}} f(x-ty)\, d\sigma(y) \right|,$$

where σ is the surface measure on the unit sphere $\mathbb{S}^{d-1}$. Stein (1976) showed that A_* is bounded in $L^p(\mathbb{R}^d)$ for $p > \frac{d}{d-1}$, $d \geq 3$ and the case of dimension two was resolved by Bourgain (1986).

In this talk, we will consider a bilinear version of the spherical maximal function defined by

$$\mathfrak{M}(f,g)(x) := \sup_{t>0} \left| \int_{\mathbb{S}^{2d-1}} f(x-ty_1) g(x-ty_2)\, d\sigma(y_1,y_2) \right|,$$

and discuss its boundedness from $L^{p_1}(\mathbb{R}^d) \times L^{p_2}(\mathbb{R}^d)$ to $L^p(\mathbb{R}^d)$ for suitable indices p_1, p_2 and p.

(The content of this talk is a joint work with Surjeet Singh Choudhary, Saurabh Shrivastava and Kalachand Shuin.)