Resistance matrices of balanced directed graphs

Let $G = (V, E)$ be a strongly connected and balanced digraph with vertex set $V = \{1, \ldots, n\}$. The Laplacian matrix of G is then the matrix (not necessarily symmetric) $L := D - A$, where A is the adjacency matrix of G and D is the diagonal matrix such that the row sums and the column sums of L are equal to zero. Let $L^\dagger = [l^\dagger_{ij}]$ be the Moore-Penrose inverse of L. We define the resistance between any two vertices i and j of G by $r_{ij} := l^\dagger_{ii} + l^\dagger_{jj} - 2l^\dagger_{ij}$. Some interesting properties of the resistance and the corresponding resistance matrix $[r_{ij}]$ will be discussed in the talk.

The classical distance d_{ij} between any two vertices i and j in G is the minimum length of all the directed paths joining i and j. Numerical examples show that the resistance distance between i and j is always less than or equal to the classical distance, i.e. $r_{ij} \leq d_{ij}$. However, no proof for this inequality is known. In the talk we will mention that this inequality holds for all directed cactus graphs. This is a joint work with Dr. R. Balaji and Professor R. B. Bapat.