Analytic Number Theory Learning Seminars
Autumn 2024

Keshav Aggarwal and Kummari Mallesham

We plan to run the following two seminar series in **Room 215**

1. Spectral Theory of Automorphic Forms on ~~**Tuesdays 4pm – 5:30pm**~~ **Mondays 11:30am – 1pm**.
2. Representation theory of Automorphic forms- Part 1 (Tate's Thesis) on ~~**Mondays 11:30am – 1:00pm**~~ **Wednesdays 11:30am – 1:00pm**.

These seminars are aimed towards Masters and PhD students interested in learning about our research areas. The initial few lectures will be delivered by us in the hope that students can take up later topics to present to each other.

Through these seminars, we aim to make the audience conversant in the basics of analytic theory of automorphic forms. This would ideally help to make current research in the analytic theory of automorphic forms accessible to the audience, with the hope of eventual fruitful collaborations and publications.

Broadly, our aim is to be able to understand various central ideas in proofs of problems regarding subconvexity bound problem, moments of L-functions, spectral theory of automorphic forms and sieve methods as introduced and developed by Iwaniec, Duke, Friedlander, Munshi, Holowinsky, Young, Michel, Blomer, Harcos, Petrow, Nelson and Jana, to name a few.

1 Spectral Theory of Automorphic forms

Main texts:

1. *H. Iwaniec*, Spectral Methods of Automorphic Forms
2. *H. Iwaniec*, Topics in Classical Automorphic Forms

Study of automorphic forms and their L-functions has been one of the central themes of modern number theory. Many powerful results have been proven by exploiting the natural connection between the geometry of the Riemann surface on which the automorphic forms live, and the spectrum of the Laplacian on this Riemann surface. Our studies are motivated by the aim to understand the following tools often used in analytic number theory:

- Petersson and Kuznetsov trace formulas
- Selberg trace formula
- Sieve methods developed by Iwaniec and his school

We will assume only basic knowledge of real and complex analysis, and start with giving a brief introduction of the upper half place, modular forms, Laplacian, Eisenstein series and Maass forms. We will then shift to studying Iwaniec's textbook.

We are open to modifying the seminar series depending on audience's knowledge and interests!

2 Tate's thesis

Main texts:

1. *D. Goldfeld and J. Hundley*, Automorphic Representations and L-Functions for the General Linear Group, Volume I
2. *J. T. Tate*, Fourier analysis in number fields and Hecke's zeta-functions

The main aim of this seminar series is to introduce the audience to the adelic language, which is very useful in generalizing classical ideas to number fields, among other applications. Our studies are motivated by trying to understand ideas around

- Ichino and Templier's Voronoi formula
- Gelbart's lectures on the Arthur–Selberg Trace formula
- Generalizing classical techniques pioneered by Iwaniec and Munshi, among many others, to the number fields setting

The seminar will run in two parts:

A. p-adic numbers and adeles over rational numbers

B. Tate's thesis over number fields

For part A, we will assume only basic knowledge of point set topology. We will start with the study of p-adic numbers, adeles and ideles, and compute various local and global integrals. We will end this part by proving Tate's thesis over the field of rational numbers.

For part B, we will assume a first course in algebraic number theory (although we will cover the basics if deemed necessary). The aim is to gain a comprehensive understanding of Tate's ideas.

We are open to modifying the seminar series depending on audience's knowledge and interests!