Curriculum Vitae

Personal Information

Date of Birth:	January 7, 1984
Place of Birth:	Mumbai, India
Nationality:	Indian
Office Address:	Department of Mathematics Indian Institute of Technology Bombay Powai Mumbai - 400076 India
Phone:	(91 9665877389)
Email Address:	srgurjar1984@gmail.com

Positions Held

2005	B.E.	Telecommunication Engineering University of Mumbai.
2007	M.Sc.	Mathematics Tata Institute of Fundamental Research, Mumbai.
2007 - 2012		Research Scholar School of Mathematics, Tata Institute of Fundamental Research, Mumbai.
2013		Long Term Visiting Scientist International Center for Theoretical Physics, Trieste, Italy

2014 - 2015	Postdoctoctoral Position Centre for Quantum Geometry of Moduli Spaces, Aarhus, Denmark
2015-Present	Faculty at the Indian Institute of Technology, Bombay, India

Research interests

Vector bundles and Principal bundles in Complex and Algebraic geometry, Complex Algebraic Surfaces, Topology.

Publications

(Roughly in decreasing order of acceptance/publishing)

1. (Joint with Nitin Nitsure)
   Harder-Narasimhan stacks for principal bundles in higher dimensions and arbitrary characteristics
   (Accepted in International Journal of Mathematics, dated 2nd Nov 2025)

2. (Joint with Indranil Biswas)
   Connections and Higgs bundles on curves defined over a number field.
   (J. Geom. Phys. 218 (2025), Paper No. 105664)

3. (Joint with Indranil Biswas)
   A Belyi-type criterion for vector bundles on curves defined over a number field
   (Accepted in Michigan Journal of Mathematics)

4. (Joint with R.V Gurjar and Soumyadip Thandar)
   Some results on the topology of algebraic surfaces.
   (Nagoya Math. J. 258 (2025), 376-397)

5. (Joint with Poonam Pokhale)
   Classification of certain affine plane curves
   (J. Ramanujan Math. Soc. 40 (2025), no. 1, 1-3)

6. (Joint with Poonam Pokale)
   Second homotopy groups of compact complex surfaces of non-general type
   (Published in Bulletin des Sciences Mathématiques; 182 (2023), Paper No. 103208, 21 pp)

7. (Joint with R.V. Gurjar and Buddhadev Hajra)
EilenbergMacLane spaces in algebraic surface theory
(Published in Geometric Dedicata; Geom. Dedicata 217 (2023), no. 2, Paper No. 31, 27 pp)

8. (Joint with R.V. Gurjar and Buddhadev Hajra)
Variation of Local Fundamental Group and First Homology Group
(Published in International Journal of Mathematics; Variation of local fundamental group and local first homology group. Internat. J. Math. 33 (2022), no. 8, 21 pages)

9. (Joint with Arjun Paul)
Criterion for existence of a logarithmic connection on a principal bundle over a smooth complex projective variety.
(Published in Annals of Global Analysis and Geometry 58 (2020), no. 3, 241-251)

10. (Joint with R.V. Gurjar and Buddhadev Hajra)
Zariski's finiteness theorem and properties of some rings of invariants.
(Published in Transformation Groups; Transform. Groups 26 (2021), no. 4, 1315-1329)

11. (Joint with R.V Gurjar)
Topological characterization of $\mathbb{C}^2$ via open algebraic surface theory
(Published in International Journal of Mathematics; Internat. J. Math. 31)

12. (Joint with Indranil Biswas)
Connections and Restrictions to Curves
(Published in Comptes Rendus Mathematique; 356 (2018), no. 6, 674678)

13. (Joint with Nitin Nitsure)
Schematic Harder-Narasimhan stratification for families of principal bundles in higher dimensions
(Published in Mathematische Zeitschrift; Manuscript Number 2016-510)

14. (Joint with Indranil Biswas, Ugo Bruzzo)
Higgs bundles and fundamental group schemes
(Published in Advances in Geometry 19 (2019), no. 3, 381388.)

15. (Joint with Nitin Nitsure).
Schematic Harder-Narasimhan stratification for Λ-modules and principal bundles
(Published in Proceedings of Indian Academy of Sciences; (2014), no. 3, 315-332)

16. **Restriction theorems for principal bundles in arbitrary characteristic.**
(Published in Journal of Algebra 426 (2015), 79-91.)

17. (Joint with V. B. Mehta).
Rationality of the instability parabolic and related results
(Published in Transformation Groups ; 20 (2015), no. 1, 99-112).

18. **Principal bundles whose restrictions to a curve are isomorphic**
(Published in Proc. Indian Acad. Sci, **121**(2011), no. 2, 165-170)

19. (Joint with Indranil Biswas).
Equivariant Holomorphic Hermitian Principal Bundles over a Rational Homogeneous Space
(Published in "Annals of Global Analysis and Geometry" 42 (2012), no. 1).

Theses Supervised

1. Buddhadev Hajra (PhD): Topology and Invariant Theory on Algebraic Varieties; 2022 (Supervised Jointly with R.V Gurjar)

2. Ketan Sutar (MPhil): M.Phil dissertation on 'A Study of Matrix Lie Groups And Their Lie Algebras'; 2022

3. Poonam Pokale (PhD): Topology of Surfaces and classification of Affine Curves;2023

4. Priyanka Magar (PhD): Smooth structures on PL-manifolds of dimensions between 8 and 10

Brief desciption of some of my work

In this section I describe my relatively recent work, roughly in reverse order in which the works were carried out.

1. Schematic Harder-Narasimhan Stratification for Principal bundles and its Consequences

(Summarizing Joint work with Nitin Nitsure in a series of 3 papers)

In a series of 3 papers (2 published and 1 accepted) we proved the following result: Let $\pi : X \to S$ be a smooth, projective morphism of Noetherian k-schemes with a relatively very ample line bundle $O_{X/S}(1)$. Let G be a reductive algebraic group over k with a choice of a Borel B and a split-torus $T \subset B$. Let E be a principal G-bundle on X. We first show that given a Harder-Narasimhan type τ, the set of points $S^\tau \subset S$ consisting points $s \in S$ such that E_s has Harder-Narasimhan type τ on X_s is a locally closed subset. We then show that if char k is zero, S^τ has a (unique) structure of a locally-closed subscheme of S with the universal property that for any S-scheme T, the base change E_T on X_T has constant Harder-Narasimhan type τ if and only if the base-change factors through S^τ. If characteristic of k is positive, we can only conclude that , the set of all relative canonical reductions of type τ of the pullback $E_T/X_T/T$ is in a natural bijection with the set of all S-morphisms from T to $S^\tau(E)$. As a consequence if char $k = 0$, for any $\tau \in \overline{C}$, the forgetful 1-morphism $Bun^\tau_{X/S}(G) \to Bun_{X/S}(G)$ is a locally closed embedding of Artin stacks, where $Bun^\tau_{X/S}(G)$ is the S-groupoid which attaches to any S-scheme T the category $Bun^\tau_{X/S}(G)(T)$ whose objects are relative canonical reductions of constant Harder-Narasimhan type τ. As τ varies over the closed positive Weyl Chamber $\overline{C}$, this defines a stratification of $Bun_{X/S}(G)$ by the locally closed substacks $Bun^\tau_{X/S}(G)$ with respect to the standard partial order on $\overline{C}$, that is, each $Bun^\tau_{X/S}(G)$ is a closed substack of the open substack $Bun^{\leq\tau}_{X/S}(G)$ of $Bun_{X/S}(G)$. The case where $G = GL_n$ and semistability considered is Giesker semistability, these results were obtained earlier by Nitin Nitsure.

2. A Belyi-type theorems for vector bundles on curves

(Joint work with Indranil Biswas)

This is work done over two manuscripts. We address the question : For a smooth complex projective curve C defined over $\overline{\mathbb{Q}}$ and a vector bundle E defined on it, when is E also defined over $\overline{\mathbb{Q}}$? For this, let X_0 be an irreducible smooth projective curve defined over $\overline{\mathbb{Q}}$ and let $f_0 : X_0 \longrightarrow \mathbb{P}^1_{\overline{\mathbb{Q}}}$ be a non-constant morphism

whose branch locus is contained in the subset $\{0, 1, \infty\} \subset \mathbb{P}^1_{\overline{\mathbb{Q}}}$ (Belyi map). For any vector bundle E on $X = X_0 \times_{\mathrm{Spec}\,\overline{\mathbb{Q}}} \mathrm{Spec}\,\mathbb{C}$, consider the direct image f_*E on $\mathbb{P}^1_{\mathbb{C}}$, where $f = (f_0)_{\mathbb{C}}$. It decomposes into a direct sum of line bundles and also it has a natural parabolic structure. We prove that E is the base change, to $\mathbb{C}$, of a vector bundle on X_0 if and only if there is an isomorphism $f_*E \xrightarrow{\sim} \bigoplus_{i=1}^{r} \mathcal{O}_{\mathbb{P}^1_{\mathbb{C}}}(m_i)$, where $r = \mathrm{rank}(f_*E)$, that takes the parabolic structure on f_*E to a parabolic structure on $\bigoplus_{i=1}^{r} \mathcal{O}_{\mathbb{P}^1_{\mathbb{C}}}(m_i)$ defined over $\overline{\mathbb{Q}}$.

We then prove analogues theorems for connections and Higgs bundles in a subsequent manuscript. In this case the curve and the vector bundle are defined over $\overline{\mathbb{Q}}$ while the connections or Higgs field is defined over $\mathbb{C}$ and we find criterion for these objects to be defined over $\overline{\mathbb{Q}}$

3. Some results on the topology of algebraic surfaces

(Joint work with R.V Gurjar and Soumyadip Thandar)

In this paper, we discuss the topology of comples varieties, focussing mainly on certain non-singular projective surfaces over $\mathbb{C}$. We first show that the fundamental group of a quasi-projective variety has negative deficiency under a certain hypothesis on its second homology, and therefore a large class of groups cannot arise as fundamental groups of varieties. For a smooth projective surface admitting a fibration over a smooth projective curve of positive genus, we give a detailed analysis of the homology and homotopy groups of their universal cover via a case-by-case analysis, depending on the nature of the singular fibers.

For smooth, projective surfaces whose universal cover is holomorphically convex (conjecturally always true), we show that the second and third homotopy groups are free abelian, often of infinite rank.

4. Second Homotopy Groups of Complex Surfaces

(Joint work with Poonam Pokale)

In a joint paper with my student Poonam Pokale, we computed the second homotopy groups of all compact, complex surfaces not of general type. The most interesting case is that of elliptic surfaces. Here our main theorem is

Theorem. *Let $f : X \to C$ be a relatively minimal elliptic fibration on a smooth projective surface X. Then :*

1. If $\chi(\mathcal{O}_X) = 0$

a) If there is atmost 1 multiple fiber, then $\pi_2(X) \cong \mathbb{Z}$ (resp. $\pi_2(X) = 0$) if genus of C is 0 (resp. genus of $C > 0$).
b) If there are 2 multiple fibers, $\pi_2(X) \cong \mathbb{Z}$ (resp. $\pi_2(X) = 0$) if genus of C is 0 (resp. genus of $C > 0$).
c) If $C \cong \mathbb{P}^1$ and there are 3 multiple fibers whose multiplicities form a platonic triple, then $\pi_2(X) \cong \mathbb{Z}$.
d) If there are 3 or more multiple fibers with the additional conditions that either $C \ncong \mathbb{P}^1$, or $C \cong \mathbb{P}^1$ and if there are 3 multiple fibers then their multiplicities do not form a platonic triple, then $\pi_2(X)$ is 0.

2. If $\chi(\mathcal{O}_X) > 0$, $C \cong \mathbb{P}^1$

a) If there are no multiple fibers or only 1 multiple fiber, then X is simply-connected and hence $\pi_2(X) \cong \mathbb{Z}^{b_2(X)} \cong \mathbb{Z}^{10+12p_g(X)}$.
b) If there are exactly 2 singular multiple fibers with d being the gcd of their multiplicities, then $\pi_2(X) \cong \mathbb{Z}^{10+12(d-1+dp_g(X))}$.
c) If there are 3 multiple singular fibers and their multiplicities form a platonic triple, then $\pi_1(X)$ is finite of order, say d and $\pi_2(X) \cong \mathbb{Z}^{10+12(d-1+dp_g(X))}$.
d) If there are 3 or more multiple singular fibers with the additional assumption that if there are exactly 3 multiple singular fibers then their multiplicities do not form a platonic triple, then $\pi_2(X)$ is infinitely generated.

3. If $\chi(\mathcal{O}_X) > 0$, $C \ncong \mathbb{P}^1$, then $\pi_2(X)$ is infinitely generated.

Using the computations on the second homotopy groups, we also classify all Eilenberg MacLane surfaces $K(G, n)$ which are not of general type. In this case n turns out to be 1 and G turns out to be torsion-free. Our theorem states :

Theorem. *Let X be a smooth compact complex $K(G, 1)$ surface of non-general type i.e., Kodaira dimension ≤ 1. Then the following assertions hold:*

1. *If $\kappa(X) = -\infty$, then X is Inoue surface.*
2. *If $\kappa(X) = 0$ then X is either Abelian Surface, Hyperelliptic surface or Kodaira surface.*
3. *If $\kappa(X) = 1$ then X admits elliptic fibration with all fibers isomorphic to an elliptic curve when taken with reduced structure. Moreover there is an unramified cover of X which is an elliptic fiber bundle over a smooth projective curve, not isomorphic to $\mathbb{P}^1$.*

5. Eilenberg MacLane Spaces in Algebraic Surface Theory

In this paper we classified all smooth, complex affine surfaces of non-general type which are Eilenberg Maclane spaces $K(G, n)$ for some non-trivial group G and some positive integer n. Again an easy observation is that n neccessarily has to be 1 and G has to be torsion-free. Equivalently, the universal cover of any such surface is topologically contractible.

The classification is by a case by case analysis depending on the logarithmic Kodira dimension $\bar{k}(X)$. Our main theorem is

Theorem. *Let X be a non-contractible smooth complex affine $K(G, 1)$ surface. Then the following assertions hold.*

1. *If $k(X) = -\infty$, then X is an $\mathbb{A}^1$-bundle over a smooth algebraic curve, not isomorphic to $\mathbb{A}^1$ or $\mathbb{P}^1$.*
2. *If $k(X) = 0$, then either $X \cong \mathbb{C}^* \times \mathbb{C}^*$ or $X \cong H[-1, 0, -1]$.*

   Moreover, if $X \cong H[-1, 0, -1]$, then there is a 2-fold unramified cover of X which is isomorphic to $\mathbb{C}^* \times \mathbb{C}^*$.
3. *If $k(X) = 1$, then X admits a $\mathbb{C}^*$-fibration with all fibers isomorphic to $\mathbb{C}^*$, if taken with reduced structure. This implies that, there is a finite unramified cover of X, which is a $\mathbb{C}^*$-bundle over a smooth algebraic curve, not isomorphic to $\mathbb{P}^1$.*

We also prove the following theorem:

Theorem. *Let X be a smooth complex projective $K(G, 1)$ surface whose universal cover is holomorphically convex. Then the universal cover of X is a Stein contractible manifold.*

6. Variation of Local Fundamental Group and Local First Homology Group

(Joint with R.V. Gurjar and Buddhadev Hajra)

In this paper we discuss various results about variation of the local fundamental group and local first homology group of normal complex spaces. At any point on a complex normal space, one can define the local fundamental group at that point. This group is trivial if the point is smooth and the dimension of variety is at least 2. For any point p on such a space X, we get a natural map from the local fundamental group at all points in a small neighborhood of p to the local fundamental groups at p. We say that the variation of local fundamental group is upper-semicontinuous if this map is injective. Our main results are:

Theorem. *Let G be a finite group acting on a smooth variety W. Let $V := W/G$ and $\pi : W \to V$ be the quotient morphism. Then the local fundamental groups satisfy upper semicontinuity property on V .*

In the above theorem, clearly the analytic local ring $\mathcal{O}_{W,q}$ is factorial for all $q \in W$, since W is smooth. In fact we prove the following stronger result which implies the above theorem.

Theorem. *Let (W, q) be a germ of normal complex space such that the local ring $\mathcal{O}_{W,q}$ is factorial and W is locally simply-connected. Let G be a finite group acting on (W, q) by complex analytic automorphisms and let $(V, p) := (W, q)/G$. Then $\pi_1^p(V)$ satisfies the upper semicontinuity property.*

As an application of upper semicontinuity of local fundamental group we will show that the singular locus of a Gorenstein complex analytic space with rational singularities has singular locus in codimension 3 at a point with trivial local fundamental group.

Theorem. *Let $\pi : (\mathbb{C}^n, O) \to (X, x)$ be a finite surjective complex analytic map of normal complex analytic germs. Assume that the local ring $\mathcal{O}_{X,x}$ at x is factorial. Then the local first integral homology group $H_1^x(X; \mathbb{Z})$ at x is trivial.*

Next we prove that the local fundamental groups of a normal complex algebraic variety are finite in number upto isomorphism. More formally, we have the following result.

Theorem. *The local fundamental groups of a normal complex algebraic variety V of dimension ≥ 2 are finite in number upto isomorphism.*

Moreover, if V is quasi-projective of degree d in $\mathbb{P}^n$ and the orders of the local fundamental groups are finite, then the orders of these groups are bounded by a function of d alone.

Along the way we proved other small intersecting results.

7. Zariski finiteness theorem

In this paper written jointly with R.V. Gurjar and Buddhadev Hajra, we gave a new proof of a special case of the Zariski finiteness theorem, namely : Let T be an affine factorial domain over the complex numbers. Let S be an inert subring of T such that the transcendental degree of S over $\mathbb{C}$ is 2. Then S is finitely generated algebra over $\mathbb{C}$.

We proved another interesting result about G_a actions namely, that if R is a regular complex affine domain of dimension 1 and suppose the additive group $\mathbb{G}_a$ acts by R-automorphisms on $R[X, Y, Z]$ such that the singular fibers of the quotient map Spec $R[X, Y, Z]$ to Spec $R[X, Y, Z]^{\mathbb{G}_a}$ are normal, then this quotient map is a locally trivial $\mathbb{A}^2$-bundle.

Along the way, we proved a stronger version of a theorem of Tyc, namely : Let $\mathbb{G}_a$ act regularly on $\mathbb{A}^n_k$ by k-automorphisms. Assume that $\mathbb{A}^n/\mathbb{G}_a$ exists as an affine variety and the induced quotient morphism is a surjection. Then $\mathbb{A}^n/\mathbb{G}_a$ is Gorenstein with at most rational singular points. In particular, $\mathbb{A}^n/\mathbb{G}_a$ has at worst canonical singularities.

8. Criterion for existence of a logarithmic connectionfor the connection or the Higgs field on a principal bundle over a smooth complex projective variety

(Joint work with Arjun Paul)

The aim of this paper was to discuss various criterion for a principal bundle on a higher dimensional non-singular projective variety to admit logarithmic connections along a divisor. For instance it is shown that : Given a holomorphic principal bundle E_H on X, where H is a connected closed algebraic subgroup of an affine algebraic group G over $\mathbb{C}$, consider the holomorphic principal G-bundle $E_H(G)$

on X obtained by extending the structure group of E_H by the inclusion mapping $H \subset G$. Then $E_H(G)$ admits a logarithmic connection singular along D if the original bundle E_H also admits a logarithmic connection singular along D. The converse implication holds when H is reductive. It is also observed that if $D \subset X$ is a reduced effective divisor in X and $\dim X > 2$, then E_G admits a logarithmic connection singular along D if and only if for some smooth hypersurface X_n in X of sufficiently large degree n, intersecting D properly, the principal G-bundle $E_G|_{X_n}$ on X_n admits a logarithmic connection singular along $D \cap X_n$.