The classification theorem:

Every finite simple group is one of the following:

1. cyclic group of prime order,
2. an alternating group,
3. a finite simple group of Lie type, or
4. one of the twenty-six sporadic finite simple groups.

Jordan (1870) published first database of finite simple groups, containing alternating groups and most of the projective linear groups, for instance, $\mathrm{PSL}_n(\mathbb{F}_p)$.

The classification is said to be complete with the monographs published by Aschbacher and Smith in 2004.

Class equation:

$$|G| = |Z(G)| + \sum_i |C_i|$$

where $Z(G)$ denotes the center of G and C_i denote the distinct non-trivial conjugacy classes in G.

Sylow Theorems:

Let G be a finite group and let p be a prime dividing the order of G. We write $|G| = p^e \cdot m$, where $p \nmid m$.

1. The group G has a subgroup of order p^e, called Sylow-p subgroup.
2. If K and H are Sylow p-subgroups of G, then $K = gHg^{-1}$ for some $g \in G$.
3. If s is the number of Sylow p-subgroups of G, then $s \equiv 1 \pmod{p}$ and $s \mid m$.

Frobenius:

A simple group of squarefree order has prime order.

Burnside:

If p is the smallest prime divisor of $|G|$ and if G has a cyclic Sylow p-subgroup P, then $G = KP$ where $K \lhd G$ and $(|K|, p) = 1$.

In particular, if G is simple, then $|G| = p$.

A finite group G is said to be **solvable**, if it has a composition series

$$1 = G_0 \triangleleft G_1 \triangleleft G_2 \triangleleft \cdots \triangleleft G_n = G$$

where the successive quotients are cyclic of prime order.

A group which is both solvable and simple is cyclic of prime order.

Burnside's theorem:

If G is a group of order $p^a q^b$, for primes p and q, then G is solvable.

Walter Feit; John Thompson:

All finite groups of odd order are solvable.

A finite group G is said to be **solvable**, if it has a composition series

$$1 = G_0 \triangleleft G_1 \triangleleft G_2 \triangleleft \cdots \triangleleft G_n = G$$

where the successive quotients are cyclic of prime order.

Finite simple groups of Lie type

Classical groups

Group	Description as a matrix group
$A_n(q),\ n \geq 1$	$\mathrm{PSL}_{n+1}(\mathbb{F}_q)$
${}^2A_n(q),\ n \geq 2$	$\mathrm{PSU}_{n+1}(\mathbb{F}_q)$
$B_n(q),\ n \geq 2$	$\mathrm{P}\Omega_{2n+1}(\mathbb{F}_q)$
$C_n(q),\ n \geq 3$	$\mathrm{PSp}_{2n}(\mathbb{F}_q)$
$D_n(q),\ n \geq 4$	$\mathrm{P}\Omega^+_{2n}(\mathbb{F}_q)$
${}^2D_n(q),\ n \geq 4$	$\mathrm{P}\Omega^-_{2n}(\mathbb{F}_q)$

Exceptional groups

$$^2B_2(\sqrt{q}) = Sz(q), \quad {}^3D_4(q), \quad G_2(q), \quad {}^2G_2(\sqrt{q}) = R(q)$$

$$F_4(q), \quad {}^2F_4(\sqrt{q}), \quad E_6(q), \quad {}^2E_6(q), \quad E_7(q), \quad E_8(q)$$

D. Gorenstein, R. Lyons, R. Solomon:
The classification of the finite simple groups

Isomorphisms (and order coincidences) among the finite simple groups of Lie type

$B_2(q) \cong C_2(q), \quad D_3(q) \cong A_3(q),$	
${}^2D_3(q) \cong {}^2A_3(q), \quad {}^2D_2(q) \cong A_1(q^2),$	
$B_n(2^m) \cong C_n(2^m),$	
$A_5 \cong A_1(4) \cong A_1(5) \cong {}^2D_2(2),$	60
$A_1(7) \cong A_2(2),$	168
$A_6 \cong A_1(9) \cong B_2(2)' \cong C_2(2)',$	360
$A_1(8) \cong {}^2G_2(3)',$	504
${}^2A_2(3) \cong G_2(2)',$	6048
$A_8 \cong A_3(2),$	20160
${}^2A_3(2) \cong {}^2D_3(2) \cong B_2(3) \cong C_2(3).$	25920

$$|B_n(q)| = |C_n(q)|,$$

$$|A_3(2)| = |A_2(4)|, \; |A_8| = |A_2(4)|.$$

Emil Artin: The orders of the linear groups,
The orders of the classical simple groups.